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Ἔχεις μοι εἰπεῖν, ὦ Σώκρατες, ἆρα διδακτὸν ἡ ἀρετή; ἢ οὐ διδακτὸν ἀλλ' ἀσκητόν; ἢ οὔτε ἀσκητὸν οὔτε μαθητόν, ἀλλὰ φύσει παραγίγνεται τοῖς ἀνθρώποις ἢ ἄλλῳ τινὶ τρόπῳ

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THE LAKATOSIAN HEURISTIC METHOD OF TEACHING AND STUDENTS' ACHIEVEMENT OF HIGHER ORDER THINKING IN GEOMETRY

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Abstract

This paper reports a main study on the effect of using the Lakatosian heuristic method to teach the surface area of a cone (SAC) on students' achievement according to Bloom's taxonomy levels. Two groups of students (the experimental and the control groups) participated in the study. The experimental group (n=98) was taught using the Lakatosian heuristic method while the control group (n=100) was taught using the Euclidean deductive method. Both groups were given a pre-test, a post-test and a delayed test. Data analysis using two-way Anova shows that both groups increased their scores *within times* (Pre-to Post and Delayed test). However, the experimental group scored higher than the control group within all times and also displayed more positive higher order thinking than the control group. It was concluded that the Lakatosian heuristic method will most likely help students to learn the SAC better than the Euclidean method.
Keywords*: Lakatosian heuristic method, higher order thinking, surface area of a cone, Cyprus secondary school*

1. Introduction

In Cyprus secondary schools, geometry teaching follows the deductive method. This method conforms to the rationalist epistemological tradition that characterizes the Euclidean theory. It promotes a lecturing teaching method which does not encourage in-depth mathematical discourse or the development of critical/creative mathematical thinking (Chazan, 1990) that it will be discussed in this section. In the Euclidean method, each step is right and leads to the proving of a theorem through rational mathematic proposals without having the students question the process. This gives the teacher the reassurance that the lesson has been "taught properly", whereas the students remain unquestioning receivers of information.

According to Sriraman & Umland (2014), Lakatos believed that the Euclidean method is “detrimental to the explanatory spirit of mathematics” (line 129 because not only may an overreliance on deduction inhibit the discovery aspect of mathematics; it might also ignore the needs of students as they learn about argumentation that constitutes a proof.

For students to develop higher order thinking and become efficient problem solvers, which are of great value in the workplace, it is essential that they are exposed to situations that enable them to think out of the box. Such exposure will enable them explore alternative views that apparently contradict their previous thinking. Mathematics is one subject that enables students to develop higher order thinking. Hence, in recent years major efforts have been made to focus on what is necessary for students to become mathematically proficient.

Mathematical proficiency is related to conceptual thinking as well as perceptual thinking, both of which are equally essential for mathematics learning (Duval, 2002). Hence, for one to solve any mathematical problem one has to undertake double analyses: mathematical analysis as well as cognitive analysis. According to Kotsopoulos (2007), “to become proficient in mathematics, students need to participate in mathematical discourses and conversations” characteristics of semi-empirical methods like Lakatosian that help students, working in group, be able to overcome their misconceptions and leading them to “foster model eliciting activity skills” (Mousoulides et al., 2007). The Lakatosian method of teaching could be a way of achieving this as it gives opportunity to students to be engaged in discourse in the classroom .

In Euclidean method, the teacher does not give the student a chance to create a hypothesis or to criticize a conjecture. Therefore, the student is not encouraged to refute the conjecture, to come up with counter-examples, or use strategies of problem solving. As a result of this, the inductive method, which requires carefully chosen examples to introduce a concept or to prove a theorem or a mathematical statement, does not exceed the deductive method. This method encourages the up-down scenario of the Euclidean axiomatic system - “A deductive system with injections of infallible truth that inundate the whole system from the top” (Koetsier, 2002, p.193).

Therefore, in the Quasi-empirical method the teacher encourages the student “to discover the solution to problems”, such as a certain proof or a certain formula, in contrast to a traditional method where a “suitably programmed Turing machine could solve [the problem] in a finite time” (Lakatos, 1976, p. 4). The teacher’s aim is to encourage students who are used to working in small groups to come up with counter-examples or use strategies of problem solving in their attempt to discover the path(s) toward the solution(s) of their problem or a conjecture. Both the teacher’s and students’ aim is

> *to elaborate the point of informal, quasi-empirical mathematics that does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvements of guesses by speculation and criticism, by the logic of proofs and refutations* (Lakatos, 1976, p. 5).

During modelling cycles involved in *model eliciting activities* skills, students are engaged in problem posing, i.e. they are repeatedly revising or refining their conception of the given problem. During the *model eliciting activities* skills, students find ways to judge strengths and weaknesses of alternative ways of thinking and whether a given response is appropriate and adequate. In this study, a *model eliciting activity* skill was used to develop a model of the SAC that described why the cone with the smaller base circle was the tallest one.

In contrast, direct method of teaching like Euclidean supports *guidance direct instructional* (Kirschner et al., 2006) which are characterizes of minimal feedback were students often become lost and frustrated, and their confusion can lead to misconceptions even more if their prior knowledge is "incomplete or disorganized" (Kirschner et al., p.84, 2006).

This study explores the effect of using the Lakatosian heuristic method to teach the surface area of a cone on students' achievement according to Bloom's (1956) taxonomy levels by answering the research question: is there any difference in students' achievement according to Bloom's taxonomy levels between the students taught the SAC by the Lakatosian heuristic method and those taught by the Euclidean one?

2. Theoretical background

This study is framed within the Lakatosian heuristic of Lakatos (1976) paradigm. Lakatos (1976) explores the contrast between Euclidean theories, such as the traditional foundationalist philosophies of mathematics, and quasi-empiricist theories that regard mathematics as conjectural and fallible (Hersh, 1978). Lakatos' main quest is summed up in the question: "what are the 'objects' of *informal* mathematical theories?" (p.150). Lakatos's argument is that mathematics, like the natural sciences, is fallible; not indubitable. Mathematical theory develops by the criticism and correction of theories, which are never entirely free of ambiguity or of the possibility of error or oversight. Starting from a problem or a conjecture, there is a simultaneous search for proofs and counterexamples. "New proofs explain old counterexamples, while new counterexamples undermine old proofs" (Lakatos, 1976).

The Lakatosian heuristic paradigm is in line with the constructivist theoretical model which believes that learning is an active process of knowledge construction where cognitive conflicts must have been engendered by the students themselves in trying to cope with different problem solving strategies (Mikropoulos & Bellou, 2013), in order to achieve higher order thinking.

3. Methodology

To undertake this study a quasi-experimental research design (Creswell, 2012) was found suitable. Two intact classes (11-grade 16-17 years old were studding optional mathematics of seven teaching periods per week) of 98 students and 100 students were used as the experimental and the control groups in different schools respectively, of the same demographic characteristics and socioeconomic backgrounds (in the same district). The experimental group was taught by the researcher using the Lakatos heuristic method (in small mixed ability groups of 5-6 students), while the control group (all classroom as a whole group) was taught by the teachers using the Euclidean method. All of the optional mathematical classes in each school were chosen. Each teacher taught in her/his classes as well as the researcher who planed in advance the teaching in each case, so as to all apply the specific teaching practices to the specific group.

The researcher maximized objectivity and minimized her involvement with the respondents during the progression of the study. This is influenced by the principles of the positivism paradigm. -The researcher was aware of the fact that she was part of the world and that posed a challenge in detaching herself from the research. Hence, to eliminate bias the research study used the quantitative approach. Quantitative research methods are deductive in nature, in the sense that inferences from tests of statistical hypothesis lead to general inferences about characteristics of a population (Harwell, 2012). The same cognitive test (Appendix A) was the main instrument of data collection and statistical analysis (which is independent of the researcher) was used to answer the research questions within time (pre- to post and delayed) in both groups.

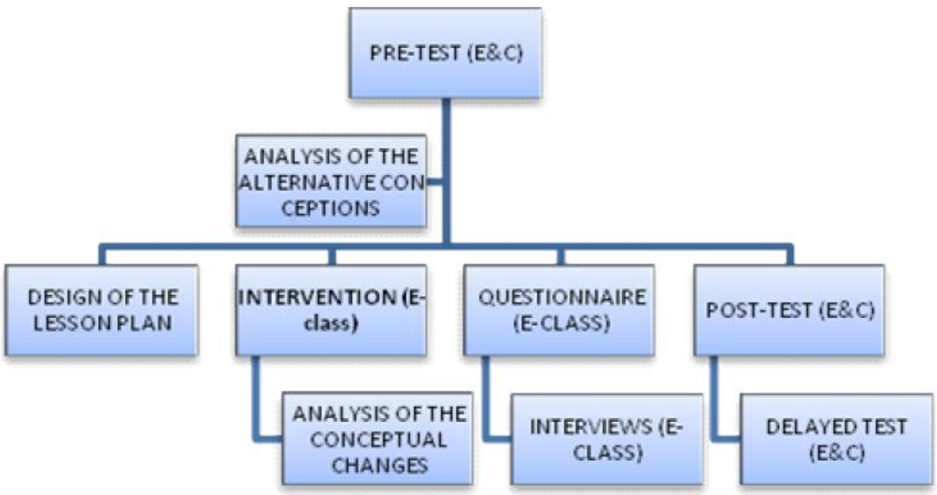
The research design of **pre-to post-test quasi-experimental design** is symbolically presented below:

N_1: O_1 X_1 O_2 Experimental Group I (Lakatosian heuristic method)
N_2: O_3 X_2 O_4 Control Group II (Traditional method)

The first row represents the experimental group while the second row is the control group. O_1 O_3 represent pre-test; O_2 O_4 represent post-tests; X_1 is the Lakatosian heuristic method used to teach the experimental group X_2 is the traditional method used to teach the control group.

Both experimental and control groups' pre-tests were examined to see if they were significantly different. It was also important to examine the pre to post tests before and after the intervention not only within the groups, but also between them.

The methodology concerning the data and the tools used in this study to examine the effect of Lakatosian method on the SAC were discussed. In particular, the process of the application in this study was developed under the following process, which signified that it was first necessary to identify students' prior conceptions through the pre-test. The questionnaire, which followed immediately after the intervention, and the interviews that ensued the week after the intervention, helped the researcher to clarify students' conceptions from alternative into scientific and they were explained according to Oh (2010) model. Therefore, the questionnaire survey and the interviews had to be refined as an auxiliary instrument.



4. The Lakatosian method

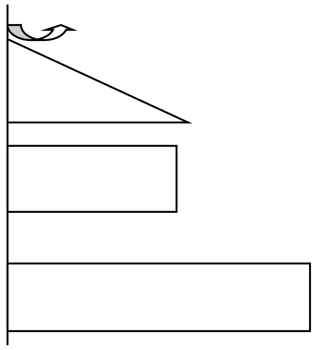
Lakatos's heuristic method as briefly explained in Hadjichristou & Ogbonnaya (2015, pp.187-188) was applied in the experimental group by using a lesson plan (of a whole teaching period) that consisted of four sections (Appendix A). In **section A, t**he teacher checked the students' pre-existing knowledge about the concept of a cone regarding the following objectives:

- Pythagoras theorem
- Elements of a circle (radius, diameter, length of arc)
- Area and perimeter of a circle and the sector of a circle
- Area of a triangle A=½absinC
- Transformation of radians to degrees and vice versa.

The teacher showed on the whiteboard the following table of two columns to check the pre-existing knowledge by matching the results in (A) column to that of the (B) column (Appendix A-task 2) giving the chance to all of the students in the classroom to react as a whole group and to give the correct answer. The teacher's role was to manipulate the students' answers and to give reflecting thinking on all concepts about the notion regarding the area of a cone as mentioned before to cover the above objectives.

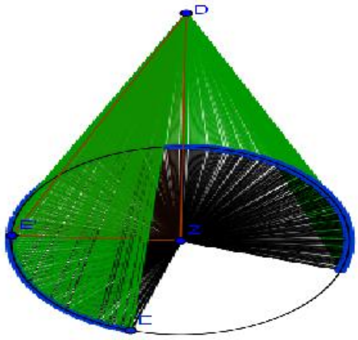
In **section B** (Appendix A-tasks 4-8) the teacher checked students' knowledge about the *notions of the surface area of a cone* constructively by checking 1) if students knew how to rotate a shape (rectangle, square, triangle) about a line and if they could name the resulting solids; 2) if students knew the generalization of the premise that if an area or a line or a point turns about the line, then it will form the volume or a curved surface area or the curve/ line, respectively. In order to cover the above objectives, the teacher used an exercise (Figure 1).

Figure 1: ***The rotation of the shapes about the vertical axis***



Also in order to cover the objectives, the teacher in addition used the mathematical applets (Figure 2) to show the second objective, developed in the GeoGebra software as well as the digital educational programme of the Cyprus Ministry of Education (2010).

Figure 2: ***A math applet in GeoGebra software about the definition of a SAC***



The role of the teacher in this section B was to declare the misconceptions/alternative conceptions between the two and the three dimensions of the constructive and deconstructive way students used regarding the SAC. By using the mathematical applet, as it is shown in (Figure 2), the lateral height (DE′) turns about the vertical axis (DZ) and students must realize the area formed–that is the SAC and they have to tell the locus of the point E-that is a circle centre Z and radius (ZE). With this activity they had to generalize the basic principle that is: if an area/line/point, turns about the axis of the rotation, then the volume/surface area/curve of revolution was formed. Also, it is important, for the students, to observe that this activity helps clarify the misconception that the SAC is formed when the *hypotenuse* (DE′) of a right angle triangle (DZE′) turns about the vertical axis (DZ) forms the curved SAC while the other vertical side (ZE) forms the base circle of a cone.

In **section C** (Appendix A-tasks 9-10) the teacher investigated the *perceptions of the students* about the construction/deconstruction of a cone from 3-dim to 2-dim and vice versa by posing the problem: *A cone hat is given. Find the material needed to make it if its lateral height is l.* The students had to imagine the shape of the sector of the cone-hat when it is developed in 2-dim and then to measure its lateral height (l) and the in-centre angle of the sector in order to find/calculate the material needed to make it. According to Herron as cited in Mulford & Robinson (2002) "their level of understanding should be extended beyond the simple ability to use words to describe the concept" (p.734). Thus, the teacher was able to realize students' understanding as well as their perceptions about the SAC from their explanations in their team work about how to construct/deconstruct a cone-hat.

Finally, in **section D** (Appendix A-tasks 11-12) the teacher examined their problem solving by posing a problem: "*A cone-hat is given. Find the material needed to make it if its lateral height is l".* The "*thought-experiment*" started by asking the students who were working in small groups, first to solve the problem and then to prove the formula (S=πrl, r=base radius and l=lateral height) of the SAC. Students were given 25-30 minutes to prove the formula and to show their presentations to the whole classroom.

For example, in their attempt to prove the SAC, the students' argumentation in the experimental group, was as follows, when they were asked:

How was the cone constructed?

By using the List as well as the video recording tapes, the following dialogue developed in the experimental group.

S1: *It's a circle!*

S2: *No! It's a sector because the circle cannot make a cone hat.*

Researcher: *Bravo this is correct answer. How do you find its area?*

S2: *We have to divide the sector in triangles [primitive conjecture], however, what will be the base of the triangles?*

After some hard thinking.

S1: *The smallest the triangles the closest the height will reach the lateral height.*

S2: *Do you mean that the height of the triangle will be the lateral height?*

(NB: she wrote down that the height of a triangle equals the lateral height of a cone (h=λ)).

S1: *Yes! So the area of the sector will be the area of the SAC.*

(NB: she found a counter example to alter S2's process of thinking).

So, the area of the sector will be $E_{sector} = \frac{®r^{2}—}{360} = \frac{®a^{2}—}{360} = \frac{®a^{2}\frac{180}{®}«^{c}}{360} = \frac{a^{2}}{2}«^{c}$

After some very hard thinking.

S2: *But, the radius (r) of the sector equals the lateral side (λ) of a cone (r=λ). What about the in-centre angle μ of a sector?*

[S3 was working silently by herself in S2's *primitive idea*, in their team]

S3: *Look! If we are adding the bases of the triangles, they are equal to the length of the arc of the sector. So,* $r\mu^{c}=\lambda\mu$.

(NB: she realized after some hard conceptualization that the radius of a sector (r) equals to the lateral height (λ) of a cone).

S2: *So my idea becomes easy* $\frac{b1+b2+...bv}{2}a = \frac{a«^{c}}{2}a$.

S3*: Yes! This is exactly the same as S2's idea. However, it's obvious that the length of the arc of a sector equals to the base circle circumference having radius ρ.*

(NB: and she wrote down the statement $\lambda\mu^{c}=2\pi\rho$).

S2: *HMm! What is the radius ρ?*

S1: *It is the radius of the base of a cone, while the circumference of the circle is 2πρ.*

S3: *Yes, by connecting the two edges [meant radius] of the sector a cone in 3-dim a cone hat is formed.*

Researcher: *Very well! Excellent! You have proved the formula.*

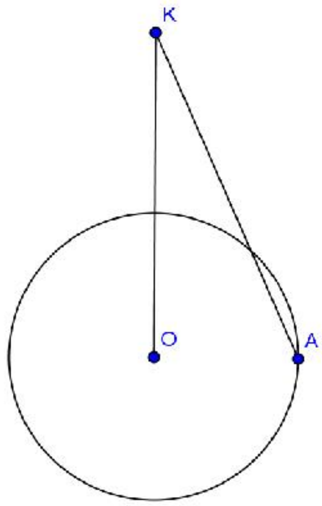
5. The Euclidean method

In this teaching method, the teacher does not give the student a chance to create a hypothesis or to criticize a conjecture. Therefore, the student is not encouraged to refute the conjecture, to come up with counter-examples, or use strategies of problem solving. As a result of this, the inductive method, which requires carefully chosen examples to introduce a concept or to prove a theorem or a mathematical statement, does not exceed the deductive method. This method encourages the up-down scenario of the Euclidean axiomatic system - "A deductive system with injections of infallible truth that inundate the whole system from the top" (Koetsier, 2002, p.193).

The Euclidean method was applied (in 20 min of one teaching period) in the control group As a direct instructional guided method is defined "as providing information that fully explains the concepts and procedures that students are required to learn as well as learning strategies" (Kirschner et al., p.74, 2006). The classroom was arranged (as a whole group) in the traditional manner, where the tables were arranged in a line; thus students were unable to collaborate with each other. After drawing a right angle triangle on the whiteboard, which was rotated on its vertical side, the teacher illustrated the properties of the cone on the whiteboard (lateral side, height, vertex, radius of a base circle) and asked students to name them. She verbally defined a solid cone and the curved surface area of a cone, by writing these definitions on the whiteboard. Then she took a cone shaped hat, opened it and by asking specific questions such as "what do you think the curved area of the cone is in the two dimensional space?" she demonstrated to students that the curved surface area of the cone is equal to the sector of the cone, after showing them its expansion. Afterwards, she took the marker and started writing the following proof on the whiteboard according to the textbook by Argyropoulos et al. (2010) as follows:

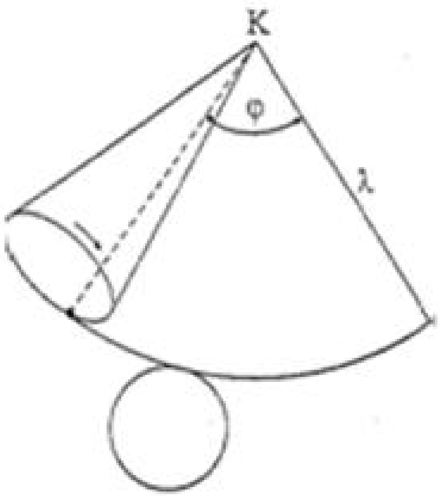
Teacher: Consider a vertical segment KO to be a plane circle and the side KA to be the hypotenuse (Figure 3) of a right angle triangle KOA with a right angle at O [the teacher first drew the side KO to be vertical on a plane circle and then she drew the sides KA and OA of the right angle triangle KOA]. The side KA is rotated around its vertical side KO to form the SAC. The hypotenuse KA upon rotation intersects a fixed point K and subtends a convex, side OA sustains a circular disk with centre O and radius OA that is the locus of the side OA which is in a plane perpendicular to KO at point O. The convex produced by the hypotenuse KA is called the lateral side of a cone. That is, the curved surface of the cone. The vertical side KO remains constant during the rotation and is termed the axis or height, while point K [is called] the peak and the circle - the locus of the side OA, is called the base of a cone having base radius OA called the radius of the cone.

Figure 3: ***Teacher's sketch to prove the SAC***



[She continues to explain the process of proving the SAC by drawing on the whiteboard indicating that it was similar to the diagram (Figure 4) in the textbook used in class]

Figure 4: ***The Surface Area of a Cone***
(Argyropoulos et al., 2010, Figure 40, p.311)



When the teacher explained how to measure the arc of the circle she was showing a drawing similar to that of the textbook, as used in figure 4, to explain that it was the same as the circumference of a base circle of a cone. Then she continued to explain students how to find the in centre angle φ of the sector.

Teacher: If we call φ the in centre angle of the sector in degrees, we have the relationship $\frac{360}{2®a} = \frac{¤}{2®°} \Leftrightarrow ¤ = \frac{°}{a}360^0$

Therefore, the developed curved surface of a cone with a side of length λ and a radius ρ is a circular section of radius λ and arc length 2πρ or, in degrees, $¤ = \frac{°}{a}360^0$.

From the above we consider that the SAC equals to S=πρλ.

[She also refers to the 2nd approach by using the limits of the area of a pyramid to prove the SAC.]

Teacher: We can also prove the SAC by using the limits of a pyramid such as: S= $\lim_{n\to\infty}(P_{base}\frac{h_{slant}}{2}) = \lim_{n\to\infty}(2®°\frac{a}{2}) = ®°a$.

[Then the teacher gave some exercises to the students until the end of the teaching period]

6. Data collection instrument and procedure

Data was collected using the same cognitive test (Appendix A) which was administered as a pre-test, post-test and delay-test. The ***pre-test*** was administered a week before the intervention. It was used to establish that both groups were at the same level of readiness. The ***post-test*** was administered two week after the intervention to obtain information on students' achievement at the various levels of Bloom's taxonomy. The ***delayed test*** was administered two week after the post test. It enabled the researchers to assess the students' retention of their learning after a period. The role of the delayed test is to provide students with the time needed to become familiar with the new concept.

The test: The test consists (Appendix A).of twelve questions (see the analysis of the test's tasks in the following section) and is divided into four sections: 1) pre-existing knowledge of the cone; 2) notion of the construction/deconstruction of the cone and the creation of its surface area; 3) students' perceptions of the construction of a cone; 4) problem solving

Interviews: The interview (Appendix C) was conducted in the experimental group only. In order to change their alternative conceptions the students needed to be exposed to discrepant events-that is, situations where their incorrect knowledge does not work (Mulford, & Robinson, 2002, p.743). The questions of the interview had a twofold goal: First, the students were asked to clarify if they changed or not any remaining misconceptions/alternative conceptions which were presented in the pre-test and then, to explain how they responded in their questionnaire. If the answers on the questionnaires concerning the pre-test as well as the questionnaire were not identical with the correct responses in the interviews, they were considered wrong in the analysis of the pre-test. The role of the researcher was not to give the correct answer to differentiate the experimental and the control group in the post-test results. The use of interviews was auxiliary. However, the role of interviews was to clarify and to support findings from other instruments such as the questionnaire and the pre-test results.

7. Analysis of the test's task according to Bloom's taxonomy

The tasks are based on the cognitive levels of Bloom's Taxonomy (Abbott, et al., 2012). However, there are no clear boundaries between levels; each level is characterized by descriptive "process verbs". The knowledge (K) level can be described by the verbs of the lowest level of cognitive skills, such as: define, label, listen, list, name, read, recall, record, relate and repeat, where this characterizes section A. The "process verbs" in the understanding (U) level include such actions as: solve, tell, describe, explain, locate, report and recognize, where this characterizes in particular the tasks of section B. The "process verbs" in the application (A) level are apply, demonstrate, illustrate and use, where these especially characterize the tasks of section C, and finally in section D the "process verbs" are mainly "calculate and solve" for the analysis level and "construct, create, design, compose" for the synthesis level.

The first section (section A) consists of the first three questions representing the Knowledge (K) level of the Bloom taxonomy. The first question is open, its aim being to identify whether the students possessed a basic knowledge of Pythagoras's theorem. The second question requires matching column A to column B; the purpose being to identify whether the students had acquired the basic knowledge of the elements of a circle (radius, area and perimeter of a circle, area and perimeter of a sector, the relationship between radians and degrees). The third question requests students to complete sentences referring to the relationship between radius and degrees.(Appendix A)

Section B, investigating *notions of the construction/deconstruction of the surface area of a cone*, consists of five questions (tasks 4-8). Three of them (tasks 4-6) belong to the knowledge (K) level. Task 7 consists of 3 parts; parts "a" and "b" belong to the comprehensive/understanding (U) level of Bloom's Taxonomy, whereas part "c" belongs to the application level (A). Its aim was to ascertain that the students knew about the geometrical meaning of the surface area and the volume of a cone (i.e. that the students were aware that volumes are formed by rotating areas and that areas are formed by rotating lines). Task 8 belongs to the application level (A) of the taxonomy; it examines the cross section of a cone. Tasks 7c and 8 are open-ended questions; whereas 7c refers to the shape that was formed in the previous tasks (7a and 7b), concerning the creation of the SAC, by rotating the hypotenuse of the right angle triangle about an axis.

Section C examines *students' perceptions of the construction of a cone. It* consists of two multiple-choice questions, one question (task 9) in the Application level of the Bloom taxonomy with the aim of exploring whether the student knows how to construct a cone from 2-dim to 3-dim and the other (task 10) in the Understanding level, aiming to identify whether the student knows how to deconstruct a cone from 3-dim to 2-dim.

The fourth section D (tasks 11 and 12), p*roblem solving,* consists of two open-ended questions in the Analysis-Synthesis (A-S) level of the said Taxonomy (see Appendix A).

8. Data analysis

Two-way Anova was used to analyse the data. This was found suitable because it enables one to measure the effect of two or more independent variables (usually an intervention) on a dependent variable (Morrell & Carroll, 2010). The Lakatosian method was applied. Group mean scores of the experimental and control groups on the tests were compared using an ANOVA (2X3) to look for significant differences. These comparisons include pre-to post test and delayed differences for both groups, a pre-test between both groups and a post-test between both groups as well as a delayed test between both groups

9. Findings

Results of the students' achievement: As shown in Table 1 (Descriptive Statistics) both groups had higher means in the post-test and delayed test than in the pre-test. Furthermore, as shown in Table 1, the experimental group had lower means than the control group in the pre- test, 11,39(±5,12) versus 12,49 (±3,91), and higher means in the post-intervention test, 16,91 (±5,93) versus 15,07 (±5,076), and in the delayed test 18,53 (±4,96) versus 14,95(±5,17).

Table 1: Descriptive Statistics
(Experimental group:1, Control group:2)

	Group	Mean	Std. Deviation	N
Pre test	1	11,398	5,12069	98
	2	12,49	3,90673	100
Post test	1	16,9082	5,93102	98
	2	15,07	5,0757	100
Delayed test	1	18,5306	4,9559	98
	2	14,95	5,17058	100

Table 2 indicates the mean achievements of each group (the experimental and the control) as regards the correct task solutions of the test (pre- to post and delayed) after grouping the questions according to the Bloom's taxonomy levels. The solutions were marked 1 for a correct answer and 0 for a wrong answer. For example, all the knowledge tasks (Tasks 1-6) of the test were added up, and they symbolize knowledge level K(1-6) is consisted of 62,5% (i.e. equals to 15/24 of the total test score) of the test. The same applies to the rest of the tasks in both the experimental and control groups. Tasks 1-3, referring to the pre-existing knowledge were included despite that both students in the experimental and the control groups achieved similar means, indicating that they had both started from the same level of readiness.

Table 2: Analysis of the Bloom's Taxonomy levels (inclusive pre-existing knowledge)

LEVELS	K(1-6)	U(7a,7b,10a)	A(7c,8,9a)	A-S(11-12)
Pre test	15marks	3marks	3marks	3marks
Experimental-group (n=98)	38,01(60,82%)	5,87(46,96%)	2,25(18%)	0,73(5,84%)
Control-group(n=100)	39,96(63,94%)	6,17(49,36%)	2,96(23,68%)	0,54(4,32%)
Post test				
Experimental-group(n=98)	52,04	9,86	7,19	5,78
Control-group(n=100)	43,88	7,25	5,33	3,17
Delayed test				
Experimental-group(n=98)	50,68	10,54	7,36	6,72
Control-group(n=100)	48,58	8,46	3,54	3,08
Test (%)	62,5	12,5	12,5	12,5

Within time scores of pre-to post and delayed tests: In this study the dependent variable refers to students' achievements within time (pre-to post and delayed) while the independent variable is the method of teaching. Within time measured each group twice (pre-to post-test and/or post-to delayed test) in examining the Bloom's taxonomy levels.

Table 3 reports the descriptive statistics of the students' achievements at these levels *within times* (pre-to post and delayed test). We are interested in the difference in the main effect, i.e. whether the main effect is significantly different within each group over time. Consequently, we wish to establish whether the students were able to achieve higher order thinking by examining the pre-to post and delayed test of the Bloom's taxonomy levels.

Figure 5: The plot of mean scores of the Groups (1: experimental and 2: control) within time (pre-to post-test and delayed test noted on the horizontal axis as 1-2-3 respectively)

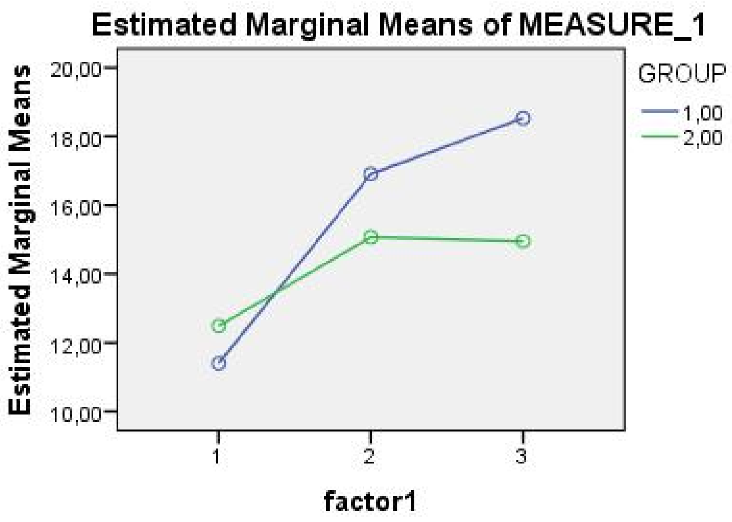


Figure 5 illustrates the plot of the mean scores of both groups. It shows how these groups performed on the pre-test, post-test and delayed test (noted on the horizontal axis as 1-2-3-respectivly), with each line representing a group. From this graph it is clear that the performance of the experimental group (Group 1), that applying the Lakatosian method was better than the control group (Group 2) where the traditional Euclidean method was applied. Even though both groups' performances were low in the pre-test, they showed increases in scores in the post-test and the delayed tests. However, the post-test and the delayed tests mean scores of the experimental group were higher than that of the control group (Figure 5) at all times (pre-to post-delayed test).

Table 3: ***Tests of Within-Subjects Contrast (pre-to post and delayed test)***

Source	factor1	Type III Sum of Squares	df	Mean Square	F	Sig.
Time	Linear	2277,24	1	2277,24	186,826	0,001
Time and group	Linear	540,329	1	540,329	44,329	0,001
Error (time)	Linear	2389,06	196	12,189		

The interaction within *groups* & *time* was also statistically significant F(1,196)=44,329, p<0,001 (Table 3) as well as the main effect *between groups* within time (pre-to post and delayed test) F(1,196)=5,829, p<0,017) (Table 4). While all groups increased their scores *within time* (pre-to post and delayed test) the increase of means scores in the experimental group was much higher than the increase of means in the control group within all times.

Table 4: ***Tests of Between-Subjects Effects (pre-to post-and delayed test)***

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Intercept	131703,4	1	131703,4	2485,578	0,001
Group	308,859	1	308,859	5,829	0,017
Error	10385,46	196	52,987		

Analysis of the Bloom's taxonomy levels within time (pre-to post and delayed test)

Given the analysis of the results of the pre-to post and delayed tests (Table 5) a significant effect *within all times* in all of the Bloom's taxonomy levels was observed, whereas a significant effect in *between groups* was *not* observed at the lower levels (Knowledge and Understanding) (F(1,196)=0,537, p=0.465 and F(1,196)=0,001, p=0.976) respectively (Table 6), as well as at the lower levels within *time & group* (F(1,196)=6,466, p=0,012 and F(1,196)=1,14, p=0,287) respectively (Table 5). This demonstrates that the Lakatosian method when compared to the Euclidean method is more effective within *all time* periods, signifying that the Lakatosian method positively affected the students' achievement at all of the Bloom's taxonomy levels, especially in the higher levels of application and analysis-synthesis.

Table 5: ***Tests of Bloom's Taxonomy levels within time (pre-to post and delayed tests) in both experimental and control groups***

Dependent Variable Pre-post test	**Source**	**TIME**	df	**Mean Square**	**F**	**Sig.**
Knowledge Level	Time	Linear	1	379,132	44,377	0.001
	Time & Group	Linear	1	55,243	6,466	**0,012**
	Error (factor1)		196	8,543		
Understand Level	Time		1	29,546	45,667	0,001
	Time & Group	Linear	1	0,738	1,14	**0,287**
	Error (factor1)		196	126,808		
Application Level	Time	Linear	1	43,12	77,622	0,001
	Time & Group	Linear	1	11,443	20,599	0,001
	Error (factor1)		196	0,556		
Analysis-Synthesis Level	Time	Linear	1	99,8	139,883	0,001
	Time & Group	Linear	1	16,163	22,655	0,001
	Error (factor1)		196	0,713		

Table 6: Tests of Between-Subjects Effects of Bloom's Taxonomy levels within time (pre-to post and delayed tests) in both experimental and control groups

Knowledge Level

Source	Type III Sum of Squares	df	F	Sig.	Partial Eta Squared
Intercept	71121,1	1	2393,82	0,001	0,924
Group	15,951	1	0,537	**0,465**	0,003
Error	5823,22	196			

Understanding Level					
Intercept	1970,75	1	1268,88	0.001	0,866
Group	0,001	1	0,001	**0,976**	0,001
Error	304,416	196			
Application Level					
Intercept	551,572	1	386,911	0,001	0,664
Group	33,363	1	23,403	0,001	0,107
Error	279,413	196			
Analysis-SynthesisLevel					
Intercept	339,171	1	234,618	0,001	0,545
Group	57,945	1	40,083	0,001	0,17
Error	283,343	196			

Extra analysis between pre-to post and post–to delayed test was considered necessary aiming to observe the main effect of Bloom's taxonomy levels within the specific time (pre-to post and post-to delayed-test).

However, the Lakatosian method had a significant effect *between the groups* in the **hot** levels (Table 6), as well as at all levels *within time* (pre-to post and delayed test) (Table 5). This was deeply supported in examined within both times (pre-to post and post-to delayed test) that the main effect was observed in the hot (application and analysis synthesis) level of the Bloom's taxonomy. We can conclude that the method may function positively at the higher levels of Bloom's taxonomy over a longer period of time, during which students can sustain their knowledge (Table 1), and that more time may be necessary to effect a change in their alternative conceptions.

Figure 6 depicts a comparison of the students' achievements at all levels of Bloom's taxonomy. The experimental group achieved better than the control group at all these levels.

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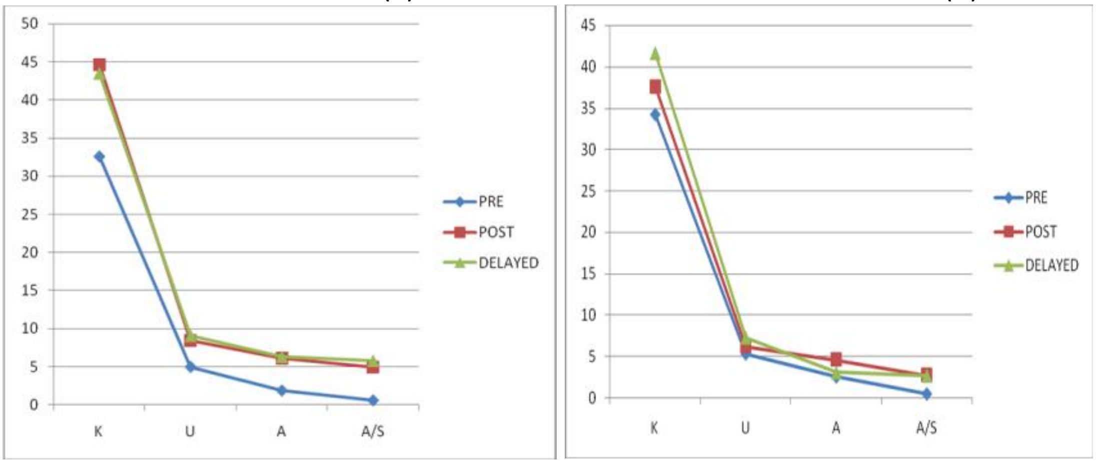
Figure 6: Comparison Analysis of the Bloom's taxonomy cognitive test results of the experimental group (Group 1) and control group (Group 2)

Experimental group (a) **Control group** (b)



Levels	K(1-6)	U	A	A-S
%	62,5	12,5	12,5	12,5

The percentage average achievement of the students at the Bloom's taxonomy levels (Table 2), as also illustrated in Figure 6, indicates that the ranges of the groups' scores in the pre-test are similar in the Knowledge level: 38,05-50,68 and 39,96-48,58 (Table 2) for the experimental and control groups respectively. However, for the post-test and delayed test the ranges were 5,87-10,54 and 6,17-8,46 in the Understanding level, 2,25-7,36 and 2,96-3,53 in the Application level and 0,73-6,72 and 0,54-3,08 in the Analysis-Synthesis level respectively (Table 2). This demonstrated that the experimental group improved more than the control group.

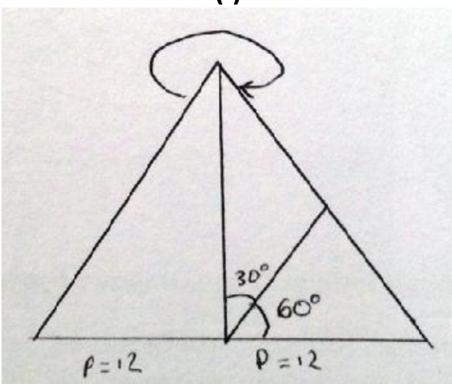
10. Discussion of the findings

As it is referred in the introduction of this article students' misconceptions are overcoming easier from pre-to post test in the experimental than the control group. The main misconception of the students in the pre-test was their inability to construct/deconstruct the cone from 2-dim to 3-dim and vice versa (see task 9 and 10 pre-post test results respectively in Appendix A). Thus, they wrongly interpreted the analysis-synthesis (A-S) data of the test task 12: *"A cone hat having a surface area the sector of a circle with in-centre angle of 60^0 and radius r=12cm is to be made using a material. Find the height of this hat"* (Appendix A), indicating inability to achieve higher order thinking in both groups of pre-test.

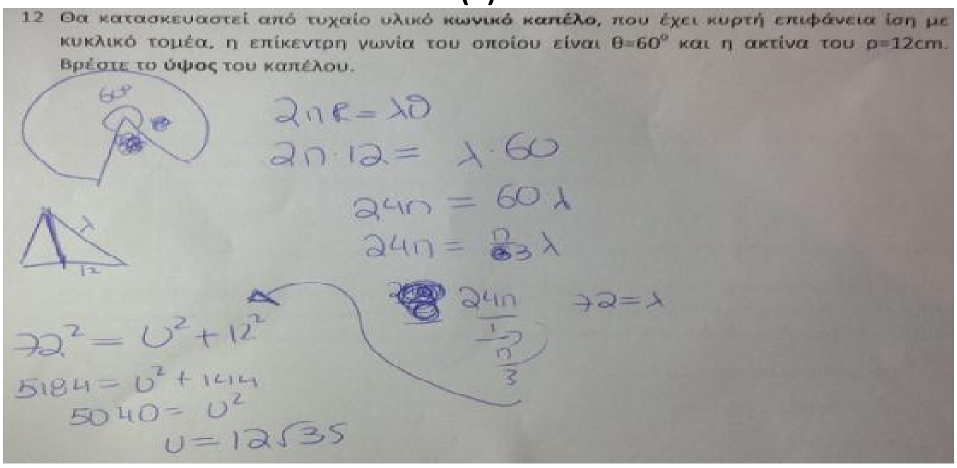
For example, student S(D) by solving the pre-test, imagined it as an isosceles triangle that rotates 180^0 about its axis (height of a triangle) to *create* the cone (misconception 1). When she tried to justify the answer of task 12, S(D) thought that the in-centre angle of the sector was the same as an in-centre angle of any shape trying to find a centre and a radius of a shape (i.e. triangle) as shown in Figure 7(I). Therefore, S(D) was wrongly led to believe that the foot of the perpendicular from the vertex of a triangle to its base was a centre of a 'circle' (in such a case triangle) with radius ρ=12. As a result, S(D) "saw" the in-centre angle as shown in the extreme case Figure 7(I) below. The great confusion that can be seen in this conceptualization shows her inability to visualize the SAC between the two spaces. As a result of this, students were not able to find relations between the two dimensions.

Figure 7: ***Misconceptions of students S(D) about the construction of a cone in task 12***

(I)



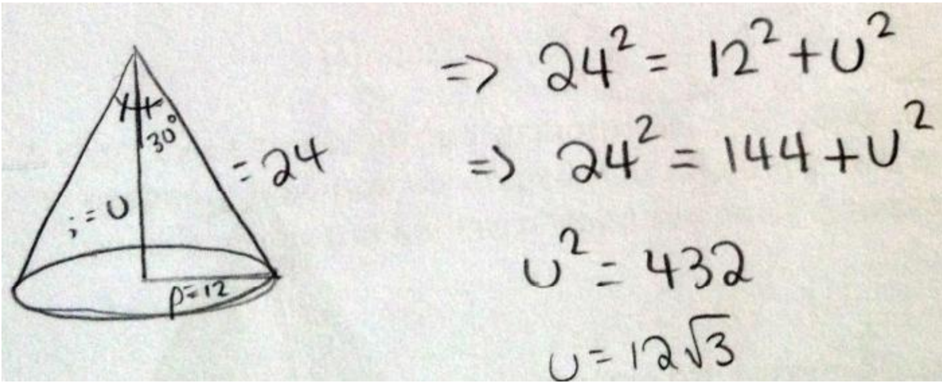
(II)



However, experimental group student S(D), due to the Lakatosian method, overcame her misconceptions as the excerpt from the post test shows in Figure 7(II). She realised how to construct/deconstruct a cone as well as how to find relations between the two spaces ($2\pi R=\lambda\theta^c$). She could transform the angle of 60^0 into the radians of π/3. However, she wrongly substituted it at the beginning in the formula ($r\theta^c$) of the arc of a circle, but afterwards she corrected it by π/3. Therefore, she realized that the radius of the sector (r) was equal to the lateral height (λ) of a cone by substituting it on the true formula of the arc of a sector as ($\lambda\theta^c$). Despite the fact that she could not solve the problem posed in the correct way, she achieved higher objectives.

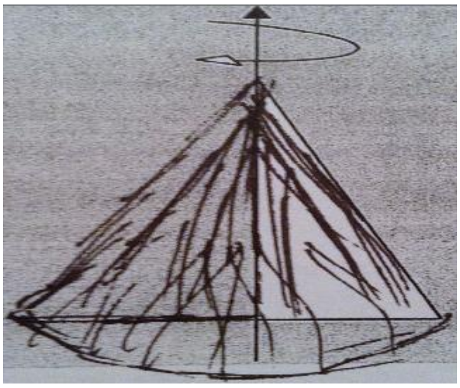
Consequently, the control group students, even in the post-test (instead of constructing a cone from a given sector) wrongly continued to draw it as a process creation by rotating 360^0 a right-angled triangle (Figure 8) about one of its vertical sides to create a cone. This misconception resulted from the method of teaching used in which they were "learning by heart" (Kotsopoulos, 2007). Therefore, they applied the theorem that the opposite side (ρ) to 30^0 is half of the hypotenuse (Figure 8). They wrongly calculated the hypotenuse as well as the cone's height by applying the Pythagorean Theorem in a right-angled triangle.

Figure 8: Misconceptions of control group on the construction of a cone



Judging by the interview (Appendix C) as well as by examining the questionnaire (Appendix B), the researcher noticed that Lakatosian heuristic method made students to acquire more "explanatory power" (Lakatos, 1970, p.137) than did the Euclidean method. For example, in the test's task 7a the students were asked: *When a right-angle triangle is turned 360^0about the vertical line, then a 3-dimensional solid is formed. Draw this solid and name it*. In the pre-test many students gave answers like *it is an isosceles triangle* by drawing a symmetrical triangle about its axis (vertical side) of symmetry with or without a base circle. Few students sketched a cone as a cross section of a cone (an isosceles or an equilateral triangle) in 2-dim arguing *it is a cone* and only one student tried to draw the locus of its hypotenuse, as a cone's surface area (Figure 9), but she couldn't name it. However, in the post-test, as well as the students' interviews, the results of the experimental group showed a *conceptual understanding* by recognising a shape of a cone in a space as well as *sequential apprehension* (Duval, 2002) that was about how to construct or deconstruct a shape, such as a cone. They also gave reasons to distinguish the solid cone from the SAC.

Figure 9: Student's art of the SAC



It is likely that the misunderstanding students do have concerning the definition of the notion of a cone and/or a SAC is due to the Euclidean teaching method which leads to a series of misconceptions (e.g. the belief that a cone, as solid cone, is *constructed* by (rotated) right angle triangles). This is exactly what many students see when a right angle triangle is rotated on one of its vertical sides. This way of imagining a cone gives rise to misunderstandings about the construction of a cone in 3-dim, as well as about the development of a cone (i.e. as a sector of a circle) on surface level (from 3-dim to 2-dim). Also, the responses of students in the experimental group included reasons such as *the angle of the sector in task 9a was greater than that in the task 9b* or *the task 9a* (Appendix A) *is the correct answer because it depends on its base radius*. Students in the experimental group improved better than the control group on task 7c: *if the hypotenuse of the above shape is turned 360^0 over the vertical line, what is the difference between the new shape and the previous solid shape?* By showing that they were realized the difference between the SAC and the solid cone.

The study has established that using the Lakatosian heuristic method to teach the surface area of a cone (SAC) has a more positive effect on students' achievements at all levels of Bloom's taxonomy than the Euclidean method. This could occur because the Lakatosian heuristic method causes students to develop deep understanding (see S(D)'s example in Figure 7) as they build new knowledge based on their previous knowledge. It allows students to progress as they justify a new concept based on prior knowledge. It, firstly, requires them to reflect on prior learning while, secondly, it allows them to explain and apply prior learning to a new scenario. It is considered to be positive for all students, not only for low achievers (see S(A)'s model eliciting activity Figure 10) but also for high ability students who are engaged in the solution of the problem

According to Harris (2010) the high ability students, particularly, can pass through the first three levels of Bloom's taxonomy quickly (see the analysis of table 5 and 6 were the first two levels of Bloom's taxonomy was not statistical significant in both group (within times or between groups). Hence, such students will need to be exposed to tasks that will be more challenging beyond these levels, in order to keep them motivated to learn.

This method not only promotes catering for the needs of students at different levels of readiness, it also causes them to participate in knowledge construction through the discourses (Moore-Russo, et. al., 2013) with which the students engage in the process of collectively finding the solution to a given problem. The finding of this study is in consonance with the view of Kulik (2003), that as students (low achievers as well as gifted students) work together in the same class, in small groups (Dhlamini & Mogary, 2013), "producing positive results and even dramatic improvements" (Kulik, 2003, p.274) in students' learning can be achieved. Such an improvement showed by the student S(A) from Iran who was motivated by the intervention in the experimental group so as to engage in the following unique problem eliciting activity. Two weeks after the intervention, when students were solving the post-test, instead of doing his test, a low achiever student S(A) tried to answer task 9 in his group working alone:

A cone-shaped tall hat is requested to be made for the junior school carnival show. Circle only one of the following shapes that is the proper one to be used for the model of the hat.

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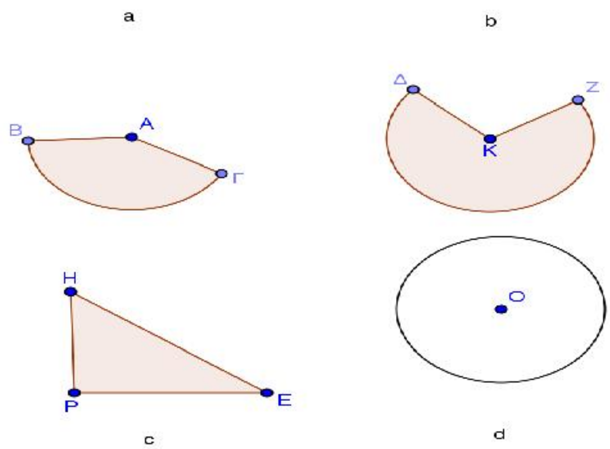
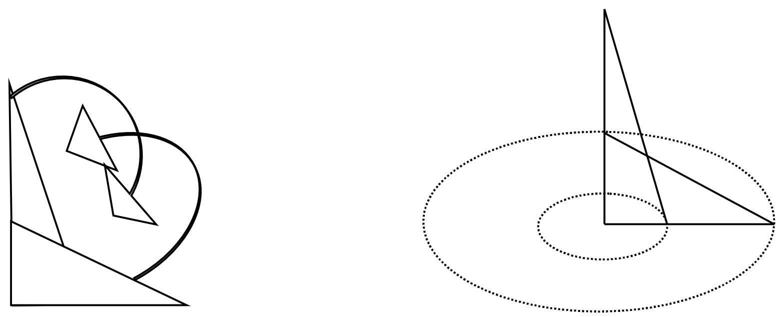


S(A) had been a low-achieving student. He was inspired to develop a real model in his attempt to solve test task 9 (Appendix A). He first thought of how to create the SAC. He did so by visualizing the cone and reasoning about what he "can see" mentally as a "mental model" (Hersh, 2014, p. 20), inspired from the thought-experiment during the intervention. With this model he explained why the cone with the smaller base circle was the tallest and he described his model.

S(A) student showed the following model in order to explain why the sector formed the tallest cone, giving a clever model justifying his explanation. He cut two pieces of congruent right-angled triangles and by putting them vertically on his desk, as shown in Figure 10a, he first transformed them and then turned them around their vertical sides (Figure 10b). He was sceptical and then said;

> *When we transform these two same* (he meant congruent) *triangles we will have different heights as well as different base radii* (Figure 10b). Then he continued saying: *By rotating them around their vertical sides we will have a cone…so the tallest cone has a smaller circumference as well as smaller area* (Figure 10b).

Figure 10: *Student S(A)'s emerging model eliciting activity skills on the SAC*



From that observation, in the post-test, they realized that the tallest hat depended also on the in-centre angle of the sector of a cone. Therefore, they could experience deductive learning (as shown in Figure 11) by justifying student S(A)'s model.

Figure 11: Student's deductive reasoning by using model eliciting activity skills

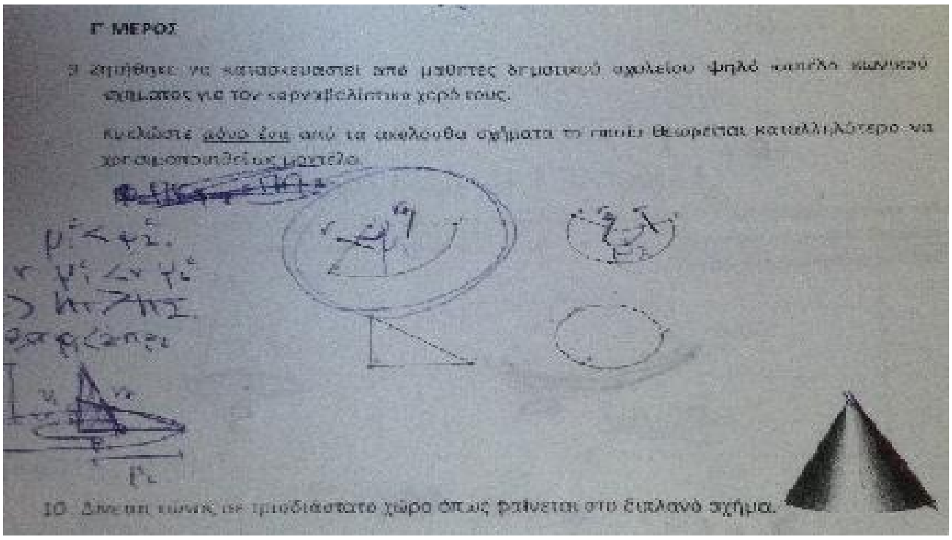


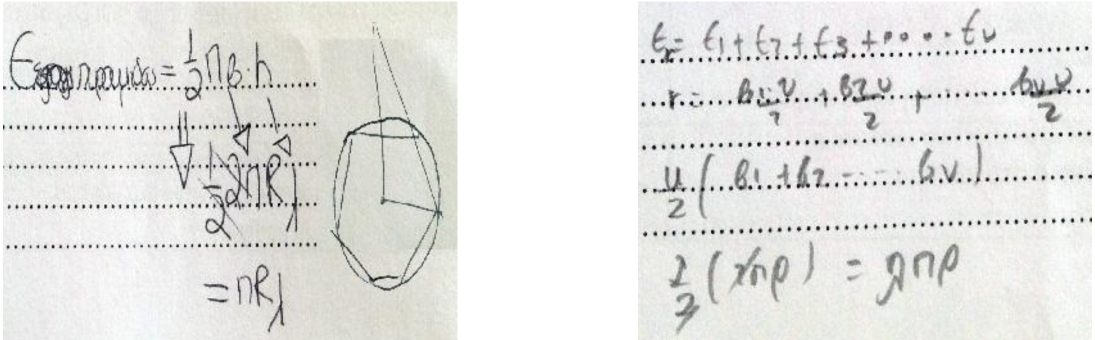
Table 2 illustrates that the average achievements of the students at the application and analysis-synthesis level in both groups and within all times were the lowest in the taxonomy levels achievements. However, the students' results on perception of the construction (task 9a & 9b in section C) of a cone were better than those of the notion of the creation of a SAC (task 7c in section B) within all times, in both groups, with the experimental group achieving higher than the control group. A great misunderstand observed in the notion of a creation of the cone graphically, when a right angle triangle is rotated about one of its vertical sides (task 7a) compared to the perception of a construction of a cone from 2-dim to 3-dim (task 9a). However, the high test results about the notion of a cone (task 7a), compare to their perception of how a cone was constructed (task 9a) might be due to a *pseudo-learning* (Vinner, 1997, p.98). This was supported by the experimental group interviews (Appendix C) as well as from their questionnaires (Appendix B) which were conducted immediately after the intervention, in both groups. They had also a great confusion among the notion of the creation of a SAC (task 7c) compared to the creation of a cone graphically (task 7a). Their perception of how the solid cone) was constructed (task 9a & 9b) or deconstructed (task 10) was also a main student' misunderstanding.

The implication is that teachers as well as students should not view *sense perception* (Hersh, 2014, p.24) as being less important than *mathematical intuition*. It is the teachers' role to use appropriate teaching methods, like the Lakatosian heuristic method, to give students opportunity to express their *sense perception* (Umland & Sriraman, 2014, line 89).

One major reason for the positive result of the Lakatosian heuristic method in this study could be the opportunity which it offered the students to *discover soluti*ons *to problems* (De Villiers, 2010; De Villiers, 2012). The said method of teaching engages them in problem solving which in turn helps them to develop creative thinking skills fundamental to development of higher order thinking.

For example, those students of the experimental group who were inspired by the first math applets used in the intervention were led to two different ways to prove the required SAC. These were by using: i) the pyramid as a solid (3-dim) and, ii) the development of the pyramid in (2-dim) as shown in the next two Figures 12a and 12b respectively.

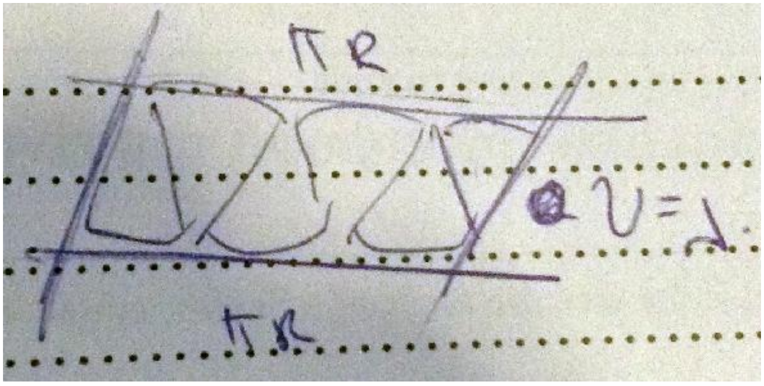
Figure 12: ***Proves of SAC from experimental group students***



a **b**

Only one student S(B) from the experimental group, in her attempt to prove the SAC, discovered that by using the cut and paste method, he could make a parallelogram. She was cutting the sector of a cone, when it is developed in 2-dim, in several equal smaller sectors. By putting them one next to the other upside down (Figure 13), she transformed it (sector) in a new shape of a rectangle (or a parallelogram). Thus, she considered the base of a parallelogram as a half perimeter of the circle (considering a perimeter of a circle equals to the arc of a sector) and the height (υ) of a parallelogram to be the radius of a sector which equals the lateral height (λ) of a cone, that is (υ=λ).

Figure 13: Proving the SAC by making a parallelogram



It is very important to mention what a student S(B) wrote in her questionnaire when asked if the lesson was interesting:

> *it is much easier to remember a formula, if you know where the maths formula is originated, and also you make yourself more able to invent formulas and solve problems in different ways. For example, when a problem cannot be solved using the traditional ways you become more able to discover new formulas to solve it*.

11. Conclusion

It has been observed that the Lakatosian method of teaching the SAC has a significant positive effect on students' achievements at all levels of Bloom's taxonomy especially at the higher order thinking (hot) level (i.e., application and analysis-synthesis) as compared to the Euclidean method of teaching. Students' achievements in the understanding level demonstrated that they were capable of achieving the highest level of the taxonomy, that of analysis–synthesis. They also did so more easily when applying the Lakatosian method, compared to when they applied the Euclidean method.

An important step for learning mathematics and for conceptual understanding in mathematical communication is necessary for ideas to become objects of reflection, refinement, discussion and amendment (Truxaw, & De Franco, 2007 as cited in Wachira, & Pourdavood, 2013, p. 5). Therefore, teachers must seek teaching methods that will help students in their conceptual learning, by analyzing both mathematical and cognitive thinking when they introduce a new mathematical concept (Duval, 2002, p. 313). In this way "learning is considered to be easier due to changes in the brain at the level of neuronal connections, and the ease with which particular synapses are activated" (Goswami, 2008, p. 264 as cited in Taber, 2009) so that students can easily find the related concepts among related tasks (such as the related tasks of this test) and to change their level of readiness in achieving higher order thinking according to Bloom's taxonomy levels.

However, teachers in the control group were graduated with honours degree, having special qualifications (master in mathematics and statistics), working in schools over 10 years, they are insisting in the same model of direct teaching as they were taught in their schools' age.

As a result, students in the experimental group interpreted the data of the tasks overcoming the visual perception, more easily than the control group, which hides two major misconceptions: i) the confusion between constructive/deconstructive a cone and creating a SAC, ii) the confusion about visualizing the SAC between the two spaces. Both led students to the inability to find relations (verbal or mathematical) between the 2-dim spaces.

Thus, visualization and the implementation in discourses in group work played an important role in teaching the SAC using the Lakatosian heuristic method. Therefore, students were able to develop cognitive activities on their own as well, observed and discovered the mathematical relations (Duval, 1999, p.7) about a concept of a SAC and between the two spaces. Such relations are deemed to be the procedures that the students developed about their *relative scientific concepts,* between *naïve scientific* and the *target scientific concept.* Through the *relative scientific concepts,* students supported their *alternative scientific concepts* so as to be led to the *final stage* of the model (Oh, 2010) which was to prove the SAC. Thus, they acquired higher order thinking skills, especially in the hot levels of Bloom's taxonomy.

Of great importance is the use of mathematical language and the sense of perception due to the Lakatosian method, in the experimental group compared to the control group.

12. Recommendations

The outcome of this study is promising; hence, we recommend that the Lakatosian heuristic method be made part of the curriculum of Geometry in Cyprus schools as it could help remedy the problems the students face in learning this subject. For this to be done effectively, teachers may need to be trained on how to effectively implement the said method.

We also recommend that this method be applied to teaching the students who study mathematics within the framework of the common core subjects; those who are not high achievers in mathematics and study it only because it is compulsory. Similarly, it should be used for teaching the students who have opted to study advanced mathematics. In this way, it will become clear whether the method yields comparatively better results when used to teach students who are low achievers or those who are high achievers.

13. Limitations of the study

The main limitation of this study concerns the number of schools and students used in the study. Only two schools took part in the study. The result would have been more convincing if the study had been conducted with a larger population of schools and students. Also, due to time constraints on the part of the participating schools, the post-test and the delayed test were administered two week and four weeks respectively after the interventions instead of *at least* two weeks and four weeks respectively after the intervention as recommended by Niaz (1998). Hence, future studies on this topic should take these limitations into consideration.

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Appendix A: The Test

The Surface Area of a Cone

Name:……………………………………………………… Class:……….

School:………. Date:…/…/…

1. It is given a right angle triangle of side's l, h, r. Find the relationship between its sides if side l is the hypotenuse of a triangle.

Answer:……………………….

2. Match the correct answers of column A with those of column B.

A circle (O, r) is given and θ^o or μ^c is the in-centre angle of a sector of the same circle with radius r and centre O.

A	B
Diameter of a circle	$2\pi r$
Area of a circle	$2r$
Area of a sector	$\frac{r^2}{2}«^c$
60^0 corresponds to	πr^2
Perimeter of a circle	$\frac{®}{3}$ radians
Length of arc	$\frac{1}{2}ab\sin C$
Surface area of a cone	$\frac{®r^2}{360}g^0$
Area of a triangle	$\frac{®rg^0}{180}$
	$r«^c$
	$®rl$

3. Complete the following sentences:

If angle $\theta=30^o$, then it corresponds to…………..radians.

If angle $\phi=\pi$,then it corresponds to ……………..degrees

If an angle is θ^o degrees , then it corresponds to ....μ^c radians

4. If a square turns 360^0over one of its sides then the shape/solid formed, will be a

(a) cylinder

(b) rectangle

(c) square

(d) rhombus

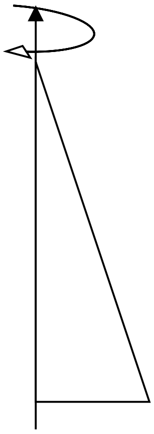
(e) ………….

5. If a right angle triangle turns 360^0 over one of its vertical sides, then the shape/solid formed, will be a…………………

6. If a line segment AB turns 360^0 over a line (ε)//AB, then the shape formed, will be:

(a) infinite lines
(b) a Surface Area of a Cylinder
(c) a symmetrical segment of AB about the line (ε)
(d) a Surface Area of a Cone
(e)……………………………

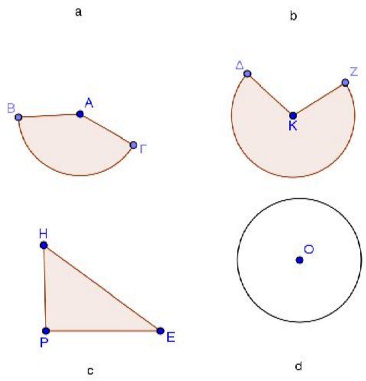
7. When a right-angle triangle is turned 360^0about the vertical line, then a 3-dimensional
solid is formed. Draw this solid and name it.



(a) The name of the solid is…………………………….
(b) What kind of triangle turns 180^0 about the above vertical line in order to form the same solid?
Answer:……………………….
(c) If the hypotenuse on the above shape is turned 360^0over the vertical side of a triangle (that is an axis of symmetry), what is the difference between the <u>new</u> and the previous solid shape?

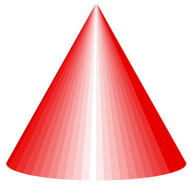
8 What is the shape of a cross-section of a cone and a plane that is passing through the vertex of the cone and the centre of its base? (Thomaides, et.al., 2000, p.349).
The shape is……………………………………….

9 What is the shape of a cross-section of a cone and a plane that is passing through the vertex of the cone and the centre of its base? (Thomaides, et.al., 2000, p.349).



The shape is…………………………………………..

10 A 3-dim cone is given in the figure below.



If we cut it by one side from the top to the base and open the shape in a 2-dimensional shape, which **one** is TRUE from the following to be the Surface Area of the Cone?

(i) A right-angle triangle
(ii) An isosceles triangle
(iii) A circle
(iv) A sector of a circle
(v) ……………………

11 An equilateral cone called the solid which is formed when an equilateral triangle side's **a** is turned of 180^0 about its height. Find: 1) the surface area of this cone 2) the side of the middle cross-section of an equilateral cone having surface area $S=2\pi cm^2$.

12 A cone-hat having surface area the sector of a circle with in-centre angle of 60^0 and radius r=12cm is to be made using a material. Find the height of this hat.
(Papanikolaou, 1975, p.368).

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Appendix B: Questionnaire

Name School:	Class: Date :
Information: This questionnaire is based on what you have learned from the current lesson. You are kindly requested to supplement it in 25-30min giving clarified and precised answers in part B. Please answer all the following questions in part B and follow the instructions in Part A.	

Part A

Instructions:

Please put x in the box next to each question which you consider to be right. If you don't know the answer, leave it blank.

Questions	Yes	No
When a right angle triangle turns over one of its vertical sides, then the solid formed, will be a cone.		
When a line segment AB turns 360^0 over a line (ε)//AB, then the shape formed, will be a cylinder.		
The Surface Area of the Cone in 2-dim is a right angle triangle.		
The Surface Area of a Cone equals to the sector having radius the lateral high of a cone		
The Surface Area of a Cone equals to πrl, where r is the radius of a base circle and l the lateral height.		
A vertical cross section of a Cone, passing through its vertex, is an equilateral triangle.		
The arc of a circle (O, l) centre O and radius l equals to lθ=2πρ, ρ= base radius of a Cone formed by this sector. and θ rad is the incentre angle of a sector of the circle radius l.		
The use of the experiment (cut and paste method) in this lesson helps you to prove the formula of the Surface Area of a Cone.		
The teacher centered lesson helps you in comprehend the lesson		

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The use of the mathematical applet/videos in teaching method contribute in better understand it.		

Part B

The lesson today has helped you to resolve your queries that concern the properties and the definition of the surface area of a cone. **Yes/No**

A) Please mark which one of the following definitions you consider more suitable to define the surface area of a cone. Put in a circle only one answer.

1. The surface area is formed by the rotation of the right-angle triangle around one of its vertical sides.
2. The surface area is formed by the rotation of the plane of a right-angle triangle around one of its vertical sides.
3. The surface area is formed by the rotation of the hypotenuse of a right-angle triangle around one of its vertical sides.

B) 1. Write down briefly what you have learned in the current lesson about the rotation of a segment about an axis of symmetry?

2. The lesson today was interesting **Yes/No**

Explain why. Give an example.

3. The current lesson has helped you to comprehend certain misunderstandings for the construction/deconstruction of the surface area of cone between 2-dim and 3-dim.

Yes/No

Explain why. What makes you solve your misunderstanding? Give an example.

4. Write down briefly what you have learnt today about the proving of the surface area of a cone, $S=\pi rl$.

Appendix C: Interviews Schedule based on the questionnaire (Experimental group)

Student's names in a group ✓ …………………………… ✓ …………………………… ✓ …………………………… ✓ …………………………… ✓ …………………………… ✓ ……………………………	Group: A, B, C, D
Introduction: The researcher will cover the following in a congenial manner: ✓ The researcher will first warmly thank students for their participation in the intervention lasting two teaching periods under video recording situations. Also the researcher will thank them for their good behavior trying to give the best effort during the intervention annoying any disturbing from the videotaping that may be doing them feel uncomfortable as their first participation in such an experience. ✓ The researcher will explain to them that she is using the voice/video recorder to capture the interview. ✓ The researcher will go through the whole information letter, drawing particular attention to the following: ➢ the student may withdraw ➢ his/her name will be kept confidential (i.e. known only to the researcher) but the researcher may anonymously quote the things she/he says. ➢ The researcher will destroy the video /audio tapes after transcribing. ✓ The researcher will ask if they have any questions ✓ The researcher will ask them to discuss the questionnaire filling after the intervention just to justify some points deeply and by explain her their way of thinking in their groups especially those who write few lines. ✓ The researcher will start to ask them one by one all the students in each of the groups spending about 10min in each one beginning from the first question by comparing what they have written in their questionnaire (the researcher will have their questionnaire in front of her have a look of their answers). ✓ Finally the researcher will thank each group.	

Follow up the questions:(start with positive feedback) The researcher wait from students to explain her what they meant exactly comparing the two methods i.e. what they mean by interesting lesson, not monotonous etc. and what made the lesson interesting for them. If they want to repeat such a lesson and why? (give points they like more /less). What make them understand the lesson than previous lessons and trying to explain her what they have learn from a lesson. Finally the researcher will ask them to explain what they write down, what they have learnt and to show her all the steps of the SAC if they remember the proves of the formula just to realize the points that remain unclear or not.	