Ἔχεις μοι ἵππην, οὐ Σώκρατες, ἄρα διδάσκον ἡ ἀρετή; ἢ οὐ διδάσκον ἅλλ', ἢ μάθητον, ἅλλὰ φύσει παραγίγνεται ἵππος ἢ ἄλλῳ τινὶ τρόπῳ.
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The scope of the MEJER is broad, both in terms of topics covered and disciplinary perspective, since the journal attempts to make connections between fields, theories, research methods, and scholarly discourses, and welcomes contributions on humanities, social sciences and sciences related to educational issues. It publishes original empirical and theoretical papers as well as reviews. Topical collections of articles appropriate to MEJER regularly appear as special issues (thematic issues).

This open access journal welcomes papers in English, as well in German and French. All submitted manuscripts undergo a peer-review process. Based on initial screening by the editorial board, each paper is anonymized and reviewed by at least two referees. Referees are reputed within their academic or professional setting, and come from Greece and other European countries. In case one of the reports is negative, the editor decides on its publication.

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EDITOR'S INTRODUCTORY NOTE

INTRODUCTION TO SPECIAL ISSUE

Behaviour of students, teachers and future teachers in mental calculation and estimation

We are happy to present the first Special Issue of our new journal “MENON: Journal for Educational Research” which was introduced in 2012. Research in Mathematics Education is a significant area of educational research, which is included in the topics of this journal.

“Behaviour of students, teachers and future teachers in mental calculation and estimation” has been chosen as the subject for this special issue on the ground of a number of reasons which are presented below.

Over the past decades, many studies have been conducted in the field of mental calculations and estimation and more precisely in relation to the definition of these concepts, the identification of the strategies used by various age groups, the relationship with other concepts, such as number sense, the procedural and conceptual understanding among others.

Many educational systems have updated the teaching of numbers and operations in mathematics, incorporating mental calculations and estimations in their elementary and middle education curricula.

Nowadays, it is considered timely to conduct research in the implementation of the teaching of mental calculation and estimation with whole and rational numbers as well as the recording of students’ behaviour and the training of pre-service and in-service teachers in these concepts.

During the last decade, researches on mental computation and estimation with rational numbers has been conducted in the Laboratory of “Nature and Life Mathematic” at the University of Western Macedonia, some of which are presented in this issue.

Most of the papers included in this issue, refer to mental calculations and estimations with rational numbers, a topic that is not very common in the literature and covers a wide age range including elementary school students, adults, as well as pre- and in-service teachers. The researches are presented according to the age range of the participants.

In their study Peters, De Smedt, Torbeyns, Ghesquière, Verschaffel distinguish between two types of strategies for subtraction: (1) direct subtraction, and (2) subtraction by addition, and provide an overview of the results of 5 related studies using non-verbal methods to investigate the flexible use of these strategies in both single- and multi-digit subtraction. Adults, students and
elementary school students with mathematical learning disabilities have participated in this research.

- **Anestakis and Lemonidis** in their study, investigate the computational estimation ability of adult learners and implement a teaching intervention about computational estimation in a Junior High School for Adults. They suggest incorporating computational estimation into Second Chance Schools and into adult numeracy teaching practices in general.

- The two papers of Lemonidis, Nolka, Nikolantonakis and Lemonidis, Kaiafa examine the behaviour of 5th and 6th grade students in computational estimation and in mental calculations with rational numbers, respectively. In these studies, the relation between students’ performance in computational estimation and mental calculations with rational number and problem solving ability are also examined.

Four studies on this issue, refer to the prospective elementary teachers’ behaviour in mental calculation and estimation.

- Anestakis and Desli examined 113 prospective primary school teachers’ views of computational estimation and its teaching in primary school. Results revealed that the majority of prospective teachers identified the importance of computational estimation for both daily life and school.

- In their research Kourkoulos and Chalepaki interviewed and examined through a test 69 pre-service teachers aiming to investigate the factors that contribute to their computational estimation ability. They found five factors that contribute to computational estimation, such as the mathematical background and the attitude towards mathematics.

- Lemonidis, Tsakiridou, Panou and Griva used interviews to examine the knowledge and the strategy use of 50 pre-service teachers in multiplication tables and their mental flexibility in two-digit multiplications by using the method choice / no-choice by Lemaire & Siegler, (1995). The results showed that prospective teachers are not flexible in two-digit multiplications and their flexibility to mentally calculate two-digit multiplications is associated with their knowledge in prep and their prep response time.

- Koleza and Koleli have used a test to study the mental computations and estimation strategies of 87 pre-service teachers. The data revealed that the prospective teachers’ number sense concerning rational numbers, basic concepts of the decimal system and elementary numerical properties was very weak.

- Lemonidis, Mouratoglou and Pnevmatikos studied 80 in-service teachers’ performance and strategies in computational estimation and individual
differences concerning their age and work experience, their attitude towards mathematics and their prior performance in mathematics during high school years.

- The last paper of Lemonidis, Kermeli and Palaigeorgiou propose a teaching intervention to sixth grade students in order to promote understanding and enrich their conceptual strategy repertoire to carry out mental calculations with rational numbers. At the same time, three teachers’ attitudes towards teaching mental computation with rational numbers, were examined.

Finally, I would like to thank all the researchers from Belgium and Greece who contributed with their papers in this thematic issue, the colleagues from the laboratory of "Nature and Life Mathematics", the reviewers of the papers and Elias Indos for the organizational and technical support in the journal.

The Editor of the first Special Issue of “MENON: Journal for Educational Research”

Charalampos Lemonidis
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COMPUTATIONAL ESTIMATION IN AN ADULT SECONDARY SCHOOL: A TEACHING EXPERIMENT

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ABSTRACT

The purpose of this study is to investigate the computational estimation ability of adult learners and to implement a teaching intervention about computational estimation in an Adult Junior High School. Adults’ estimation skill is measured through interview, before and after the intervention, with a researcher-designed tool. A questionnaire is also used to evaluate their attitudes towards mathematics. The math teacher who implemented the teaching experimental is interviewed too. The results suggest that, although adults could estimate to some extent, their estimation skill was significantly improved due to the teaching intervention. Factors that may affect estimation skill, such as prior knowledge of strategies and involvement in everyday activities with measurements or mental calculations, are also investigated. Furthermore, math teacher’s participation in this research may have led to a PCK expansion, even though her attitude towards estimation was neutral. Finally, educational implications are discussed too.

Keywords: computational estimation, adult learners, teaching experiment.

1. INTRODUCTION

Literature about estimation has covered many topics mainly focusing on computational estimation ability. As expected, most research findings concentrate on students’ strategies and performance. It seems that performance and estimation strategies can differ depending on educational settings or on culture. It is also known that estimation ability improves with age (Lemaire & Lecacheur, 2002). Although many papers investigate adults’ estimation ability as well, most samples involve teachers. Only few studies examined computational estimation ability of adult learners (e.g. research papers with undergraduate students). As found, adults are not necessarily proficient estimators. As a matter of fact, high ability in exact calculations is not in alignment with their performance in numerical situations which require estimation (Lemaire, Arnaud & Lecacheur, 2004. Hanson & Hogan, 2000).
The principal idea for the design of the present study was to investigate the computational estimation ability of adult learners, since most research papers about estimation concern students or teachers. We strongly suggest that estimation is more useful for adults since they deal with money and transactions every day. This is why adults’ participation in an intervention related to estimation ability was another important objective of our research. Thereby, it can be examined if instruction is really beneficial for adult learners and if there are any specific boundaries or advantages that should be taken into account. Consequently, an adult secondary school appealed to us as an inspiring educational setting to implement our study.

2. THEORETICAL BACKGROUND

Mental calculations have been incorporated into the curriculum with the intention of introducing a number sense approach, since written computations had a long dominance in mathematics education before (McIntosh, 2004). Likewise, computational estimation has been included due to its practical utility for predicting or checking a result (Segovia & Castro, 2009) apart from its benefit for the development of number sense. Therefore, mental procedures can produce either an exact or an approximate arithmetic outcome depending on the situation and the purpose of the mental operation.

Computational estimation has been thoroughly investigated and many factors affecting computational estimation have been found. First of all, number type or operation type of problems can influence computational estimation skill. Tsao & Pan (2011) found that students perform better in estimation with natural numbers and worse in problems with fractions. Tsao (2013) documented elementary teachers’ low performance on estimations involving multiplication and division. These difficulties suggest that a deep understanding of rational numbers and multiplicative reasoning is necessary for estimation and imply a strong relationship between estimation and number sense, which includes knowledge of and facility with place value, numbers and operations.

Afterwards, cultural factors, such as national educational systems or social values about mathematics, can influence students’ performance and attitudes towards estimation (Liu & Neber, 2012). Imbo & LeFevre (2011) also reported differences between different ethnic groups: Chinese selected computational estimation strategies less adaptively than the Belgians, although they were faster and more accurate. The writers explained this finding in terms of previous educational experience and attitudes towards estimation (Asians prefer exactness).

Next, attitudes can influence the value credited to estimation. Tsao (2013) found that pre-service elementary teachers hold computational estimation in high regard. However, their attitudes were neutral in general, because they rarely applied estimation strategies and, as a result, had insufficient experience in computational estimation. Except for the teachers, some learners may also feel uneasy about
estimation, since answers in school mathematics are expected to be “right” or “wrong” and must be exact (Newmarch & Part, 2007).

Last but not least, age is crucial for the development of estimation skills. Longitudinal studies showed that computational estimation skill develops with age, given that it depends on learning number facts and problem-solving procedures, such as rounding (Booth & Siegler, 2006). Psychological research with executive functions aligns with previous studies demonstrating that strategy selection and execution in computational estimation tasks improve with age (Lemaire & Lecacheur, 2011). Finally, Lemaire, Arnaud and Lecacheur (2004) documented age-related strategic changes in computational estimation performance, as older adults provided less accurate estimates than younger ones, although both groups had similar strategy preferences.

Considering that adults can draw on their everyday experience and judgments (e.g. are 4€ enough to buy a bread and a carton of milk?), they should be expected to be fluent estimators. However, research does not report such facility with estimation. For instance, Hanson & Hogan (2000) found that adults’ estimation skill was much worse than their computational skill. Their preference to exact calculations can be probably interpreted as a result of limited prior experience in computational estimation in formal education.

It is surprising, though, that most research focuses on students or on teachers and only few recent studies about estimation concern adults. In fact, it could be claimed that estimation is more necessary, useful and beneficial for adults, because they deal with money and transactions all the time. In addition, adults are at advantage because they can easily utilize their daily activities in order to learn estimation strategies, while young students may have some trouble. Indeed, Yang & Wu (2012) documented students’ difficulty in contextual computational estimation problems, because they needed to transform verbal words into number symbols. On the other hand, adult learners can engage in contexts with time and money in order to learn strategies and see the value of estimation (Ness & Bouch, 2007).

On this account, computational estimation should be considered to be introduced in adult numeracy programmes. Lemonidis (2010) proposed the incorporation of estimation in the teaching practices of mathematicians in Greek Second Chance Schools, as it constitutes an integral part of number sense and operations. Second Chance Schools offer a two-year intensive programme for adults and issue their graduates with a leaving certificate equivalent to a Junior High School certificate, which is typically awarded upon the completion of compulsory education. Having completed primary education is the only requirement for the enrolment.

The aim of this study is to investigate adult learners’ computational estimation skills in a Greek Second Chance School and to implement a teaching experiment with a series of computational estimation activities. The present study focused on students’ performance and mental strategies with the intention to explore factors that can affect estimation skill. Additionally, the feasibility of teaching computational
estimation in such schools was tested with the scope of introducing mathematical topics that are meaningful, useful and beneficial for adult learners.

3. METHOD

Participants. Fifteen adults (10 males), aged from 19 to 65 (mean age 33 years and 3 months) participated. Four of them come from Albania. They all have a primary school leaving certificate and were attending the first year of a Second Chance School in North Greece during this study. One of them is a pensioner, six (all males) were working and the remaining eight were unemployed at that time.

The teacher. The teacher who engaged in the professional learning intervention and conducted the experimental teaching sessions is a mathematician with a M.Sc. in theoretical calculus. She worked as a supply teacher in secondary education for the last four years, although her studies did not include any subjects about mathematics education and neither had she a further training in teaching maths.

Preparing for the experiment. The researcher contacted the maths teacher and observed her lessons for five weeks. His intent was to meet the participants in their original learning ecology and to estimate their skills in mathematics in order to ensure that the experiment would correspond to their needs. That is why he noticed the tasks they were typically asked to solve and the materials they used, the kinds of classroom discourse and the norms of participation.

As observed, the instruction was mostly teacher-centred. The mathematician usually used school textbooks as educational resource and distributed photocopies with solved examples of numerical problems. The copies were also accompanied by a worksheet of tasks. The vast majority of the tasks included pure numerical problems. Adult learners were asked to solve them on the whiteboard and, when needed, she demonstrated the steps required for the solution. Most interactions focused more on the procedural and less on the conceptual understanding. No classroom technology tools were used.

Provided that the researchers did not intent to change the classroom teacher’s practices, they designed three worksheets with tasks. The design of the activities was consistent with Van den Heuvel-Panhuizen’s (2001) learning-teaching trajectory. The first worksheet was about rounding (e.g. [census figures are provided] how would you indicate the number of residents in an information booklet of our local prefecture?). The second included estimation problems with addition and subtraction (e.g. Jacob & Martha want to buy these products [...] – are 30€ enough?). The last worksheet was about estimation with multiplication and division (e.g. can you help Sophie note the decimal point in the following operation?). The designed activities were adjusted to the local educational setting. For example, fractions were excluded from the estimation tasks that were going to be used as material in the forthcoming experimental teaching sessions.
Two changes were adopted in the teaching practices. First, the estimation worksheets contained many contextual problems. This direction was in accordance with the nature of estimation and with the needs of an adult numeracy instruction. Second, adult learners were allowed to use calculators during the instruction.

Furthermore, the teacher participated in a two-hour professional learning intervention (organized by the researcher) about computational estimation and its teaching. Among other things, the concept of estimation, common strategies and its necessity in mathematics and in mathematics education were discussed. Eventually, she also familiarized with the activities designed for the teaching intervention.

**Conducting the experiment – Data sources.** Many methods were used to collect data: pre- and post-tests, questionnaires, observation and teacher’s interview.

Students’ computational ability was examined through individual interviews before and after the teaching intervention (pre- and post-test respectively). Interviews took place in school and lasted approximately 15 minutes each. For that reason, an instrument with 9 questions was developed by the researchers, adapted to the students’ needs. As a result, it included only one- or two-digit numbers. Four tasks regarded estimation in addition and subtraction (e.g. *Harry bought an electric shaver. Its initial price was 69€ but he bought it for 37€. The discount was over or under 40€? Explain*.). The other five tasks included numerical situations which required estimation in multiplication or division (e.g. *Zoe, a primary teacher, bought 67 candies for her 10 students. Were there enough candies to take 7 each? Explain*.). Each pre- and post-test task had the same numerical data but the context slightly changed.

A questionnaire with 12 items on a four-point Likert-type scale (-2=disagree a lot, ..., 2=agree a lot) was used to measure students’ attitudes toward mathematics. Reverse scoring was also used for statements in negative terms. The items focused on three aspects of attitudes toward mathematics:
1. confident (e.g. *I am good at mathematics*),
2. like learning (e.g. *mathematics is boring*),
3. value (e.g. *mathematics is not useful for most occupations*).

The categories were in alignment with TIMSS 2011 context questionnaire scales (Martin & Mullis, 2012) but were slightly adapted for adults.

The teaching experiment lasted for three weeks (7 lessons particularly). Its conduction began a week after pre-test and attitudes measurement. The first author was present during the teaching sessions. A witness is necessary, according to Steffe & Thompson (2000). Field notes were recorded through observation.

A semi-structured interview was conducted with the maths teacher after the teaching intervention with the view to exploring her ideas and beliefs about computational estimation and her teaching experience.
4. RESULTS

4.1. Performance on the 9-task tests (N=13)

Students’ responses were classified into categories as shown in Table 1. Mental responses, either given by an exact mental calculation or by a computational estimation, represent the majority of their answers (58.1% and 82.1% in the pre- and post-test respectively). It is obvious that most mental responses resulted from computational estimation strategies. Responses based on written algorithms or on pocket calculator were less common in both tests. Additionally, there were some correct responses that could not be sufficiently explained by the participants (categorizes as “vague explanation”). Finally, about one fifth of the responses in the pre-test were wrong. However, the mean score of the participants’ errors was almost reduced to the half in the post-test.

Table 1: Frequency of mental responses by type per test

<table>
<thead>
<tr>
<th>Response</th>
<th>pre-test Frequency</th>
<th>Mean (%)</th>
<th>post-test Frequency</th>
<th>Mean (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mental responses (total)</td>
<td>68</td>
<td>58.1</td>
<td>96</td>
<td>82.1</td>
</tr>
<tr>
<td>Computational estimation</td>
<td>41</td>
<td>35.0</td>
<td>71</td>
<td>60.7</td>
</tr>
<tr>
<td>Exact mental calculation</td>
<td>27</td>
<td>25.0</td>
<td>25</td>
<td>21.4</td>
</tr>
<tr>
<td>Written algorithm</td>
<td>9</td>
<td>7.7</td>
<td>2</td>
<td>1.6</td>
</tr>
<tr>
<td>Vague explanation</td>
<td>6</td>
<td>5.1</td>
<td>3</td>
<td>2.6</td>
</tr>
<tr>
<td>Pocket calculator</td>
<td>11</td>
<td>9.4</td>
<td>3</td>
<td>2.6</td>
</tr>
<tr>
<td>Wrong answer</td>
<td>23</td>
<td>19.7</td>
<td>13</td>
<td>11.1</td>
</tr>
</tbody>
</table>

According to the preceding data, it can be assumed from the pre-test that the adult learners are capable of doing mental calculations and computational estimations, although they had not previously received an explicit instruction in this field. Afterwards, the results from the post-test suggest that the students’ mental ability was significantly improved after the teaching intervention, since mental responses increased and all the other scores were reduced.

As shown in Table 2, a performance by type of operation analysis showed that estimation in addition or subtraction was easier than estimating in multiplication or division in both tests. Additionally, the mean score of mental responses increased in both categories and the gap closed. This implies that the teaching of estimation led to a deeper understanding of operations.

Table 2: Mean score of mental responses by operation type per test

<table>
<thead>
<tr>
<th>Operation type</th>
<th>pre-test</th>
<th>post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (%)</td>
<td>Mean (%)</td>
</tr>
<tr>
<td></td>
<td>(tasks)</td>
<td>(tasks)</td>
</tr>
<tr>
<td>Addition-subtraction</td>
<td>65.4 %</td>
<td>86.5 %</td>
</tr>
<tr>
<td></td>
<td>(2.62 out of 4</td>
<td>(3.46 out of 4</td>
</tr>
<tr>
<td></td>
<td>tasks)</td>
<td>tasks)</td>
</tr>
<tr>
<td>Multiplication-division</td>
<td>52.3 %</td>
<td>78.5 %</td>
</tr>
<tr>
<td></td>
<td>(2.62 out of 5</td>
<td>(3.92 out of 5</td>
</tr>
<tr>
<td></td>
<td>tasks)</td>
<td>tasks)</td>
</tr>
</tbody>
</table>
Next, the question is whether an improvement was noticed for all participants. As a matter of fact, the individual performance of every student was improved due to the teaching experiment. Participants’ mental responses per test are presented in Figure 1, in which pre-test mental performance scores are presented in descending order with S1 having the best mental performance and S13 the worst.

**Figure 1:** Frequency of mental responses by student per test (max=9)

As shown above, the 7 lessons about computational estimation were really beneficial for the thirteen students, since each had a better score in the post-test. Furthermore, their improvement was evident not only in their performance, but also in their wrong answers and their mental strategies as well.

Regarding the wrong responses, they were reduced from 23 (19.7% of total answers) to 13 (11.1%). Most incorrect answers were given at random or could not be sufficiently explained. The error types are presented in Table 3. It can be strongly suggested that the teaching experiment eliminated students’ errors.

**Table 3:** Frequency of error types per test

<table>
<thead>
<tr>
<th>Type of error</th>
<th>pre-test Frequency</th>
<th>Mean (%)</th>
<th>post-test Frequency</th>
<th>Mean (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrong operation</td>
<td>1</td>
<td>0.9</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>Error in calculation</td>
<td>9</td>
<td>7.7</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>Irrational explanation</td>
<td>6</td>
<td>5.1</td>
<td>4</td>
<td>3.3</td>
</tr>
<tr>
<td>Random answer</td>
<td>7</td>
<td>6.0</td>
<td>7</td>
<td>6.0</td>
</tr>
</tbody>
</table>

As already demonstrated above, it has been found that the correct responses with vague explanation decreased from 6 to 3 (see Table 1) and that wrong answers with irrational explanation were reduced from 6 to 4 (see Table 3). It can be assumed that a long-term practice with mental calculations and computational estimations could enhance number sense and improve metacognitive skills in the long run, similarly to the metacognitive improvement noticed in an experimental design research conducted by Bobis (1991).
An analysis of errors by type of operation showed that the participants had difficulties in estimation involving multiplication or division (see Table 4). The mean scores of errors decreased for all operations after the implementation of the teaching sessions. The improvement was bigger in estimation problems of multiplicative reasoning.

**Table 4: Mean score of errors by operation type per test**

<table>
<thead>
<tr>
<th>Operation type</th>
<th>pre-test</th>
<th>post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11.5 %</td>
<td>7.7 %</td>
</tr>
<tr>
<td></td>
<td>(0.46 out of 4)</td>
<td>(0.31 out of 4)</td>
</tr>
<tr>
<td>Addition-subtraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication-division</td>
<td>26.1 %</td>
<td>13.9 %</td>
</tr>
<tr>
<td></td>
<td>(1.31 out of 5)</td>
<td>(0.69 out of 5)</td>
</tr>
</tbody>
</table>

Concerning students’ mental responses, their mental strategies are presented in Table 5. Exact mental calculations were frequently used in tasks with integers, while computational estimation strategies were mostly used in tasks with decimals. Rounding was the most common computational estimation strategy (21 times in pre-test, 50 in post-test). Front-end strategy was also frequent. Ultimately, other strategies were observed too.

**Table 5: Frequency of mental strategies per test**

<table>
<thead>
<tr>
<th>Mental strategy</th>
<th>pre-test</th>
<th>post-test</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact mental calculation</td>
<td>30</td>
<td>26</td>
<td>-4</td>
</tr>
<tr>
<td>Rounding</td>
<td>17</td>
<td>43</td>
<td>+26</td>
</tr>
<tr>
<td>Front-end strategy &amp; compensation</td>
<td>6</td>
<td>13</td>
<td>+7</td>
</tr>
<tr>
<td>Rounding &amp; compensation</td>
<td>4</td>
<td>7</td>
<td>+3</td>
</tr>
<tr>
<td>Front-end strategy</td>
<td>8</td>
<td>3</td>
<td>-5</td>
</tr>
<tr>
<td>Substitution</td>
<td>3</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>Averaging</td>
<td>-</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td>Compatible Numbers</td>
<td>-</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>68</strong></td>
<td><strong>96</strong></td>
<td><strong>+28</strong></td>
</tr>
</tbody>
</table>

As a result of the teaching intervention, a significant increase of mental strategies was found. Particularly, the frequency of rounding strategy was more than doubled. Compensation strategy use was also doubled in post-test. It is possible that this increase is due to teacher’s tendency to insist on the rounding rule and to ask for accuracy. Nevertheless, she utilized situation based rounding as well, in order to highlight the importance of the word problems’ context, though less frequently.

As can be seen, averaging and compatible numbers strategies were noticed incidentally in post-test. Despite the fact that the tests included tasks that facilitated the use of these strategies, the latter were not recorded more than once. This is because of the maths teacher’s omission to refer to any of these strategies in class, although she was asked by the researchers to teach them explicitly.
4.2. Performance of students with difficulties in math

Two participants were examined with 6-task tests that were easier than the tests used for the rest 13 students. Many reasons contributed to this decision. First, their difficulties in learning were observed by the first author during the preparation of the study. Second, the maths teacher insisted that an easier instrument should be used for their interviews. Third, they were both reluctant to participate in the study because of their low confidence. Finally, the psychologist of the school confirmed that they both had low confidence and difficulties in learning but stated that they had not been checked for specific learning disorders, though this possibility cannot be rejected.

As shown in Table 6, the first student (S14) preferred to give random answers and to use the pocket calculator. In spite of its use, her responses did not make any sense, since she repeated the same wrong patterns. For instance, in task 6 she could not calculate how much should each of five persons pay if the total debt was 51.34 €. In fact, her random answers were much bigger than the total amount in both tests. The other student (S15), who did give a right answer to the easier tasks, was unwilling to use the pocket calculator. On the contrary, she responded at random.

<table>
<thead>
<tr>
<th>Task</th>
<th>pre-test</th>
<th>S14</th>
<th>S15</th>
<th>post-test</th>
<th>S14</th>
<th>S15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task 1</td>
<td>4.75+4.65 &gt; or &lt; than 10?</td>
<td>Random answer</td>
<td>✓ Front-end strategy</td>
<td>1. Random Answer</td>
<td>4+75=79 and 4+65=69</td>
<td></td>
</tr>
<tr>
<td>Task 2</td>
<td>2 + 7 + 4 + 6 + 3 + 6</td>
<td>1. Random answer 2. Pocket calculator: 2+7=9, 4+6=10, 3+6=9, could not decide if it’s equal to 9 or 10</td>
<td>✓ Exact mental calculation</td>
<td>Pocket calculator: 2+7=9, 4+6=10, 3+6=9, decided it’s 10.</td>
<td>✓ Exact mental calculation</td>
<td></td>
</tr>
<tr>
<td>Task 3</td>
<td>7.05+6.89 +7.12+6.75 +7.35</td>
<td>Random answer</td>
<td>Random answer</td>
<td>✓ Pocket calculator: around 39</td>
<td>Random answer</td>
<td></td>
</tr>
<tr>
<td>Task 4</td>
<td>9 : 5</td>
<td>1. Random answer 2. Pocket calculator: 1+1=2, 1+1=2, 1+1=2. So it’s 6.</td>
<td>✓ Around 2, could not explain why.</td>
<td>Pocket calculator: 0.20 €</td>
<td>✓ Around 2, could not explain why.</td>
<td></td>
</tr>
<tr>
<td>Task 5</td>
<td>3 x 28</td>
<td>Random answer</td>
<td>Random answer</td>
<td>✓ Pocket calculator: 28 x 3 = 84.</td>
<td>Random answer</td>
<td></td>
</tr>
</tbody>
</table>
4.3. Factors affecting computational estimation skill

Prior Knowledge & Experience

The importance of prior knowledge and experience was prominent in this study. Actually, even the data sources varied in accordance to the cognitive level of the participants provided that two different instruments were used depending on the target sample. Obviously, the students with difficulties in mathematics had not memorized as many arithmetic facts as the rest participants. S14 needed a calculator even for one-digit numbers addition. Additionally, it can be claimed that they did not possess variety of procedures for solving the estimation problems. As a matter of fact, they did not even cultivate their rounding skill after the teaching intervention, since there was no significant improvement in most cases. It is to be questioned, however, whether their bad performance is a result of low number sense or whether it is due to mathematical anxiety.

Afterwards, as shown in Tables 2 & 4, the type of operation had a strong effect on participants’ mental performance, since estimating products or quotients was found to be more difficult than estimating sums or differences. As can be seen in Figure 2, in both tests the adult learners demonstrated higher mean score on estimation tasks with addition or subtraction in comparison to problems with multiplication or division.

Figure 2: Mean (%) of mental responses by operation type per test

This finding implies that estimation skill depends on computational skill or on number sense in general. This is why good estimators are expected to be fluent in operations or to have a good number sense. Such a hypothesis was tested in the post-test interviews.

Indeed, personal information about the four participants with the highest performance showed that the best estimators had a strong cognitive background. In fact, the best estimators engaged in everyday activities that required mental procedures, like calculating mentally or estimating measures. It is suggested that their prior knowledge and experience was critical for estimating.
First of all, a surprising relationship between computational and measurement estimation is implied. Both participants with the highest mental performance (S1 & S2) measure lengths and surfaces all the time at work. Particularly, their jobs involve measurements of plasterboards (S1) and of tiles (S2). Consequently, they are both considered to be good at measurement estimation, since fluency in measuring is apparently necessary for installing plasterboards or floorings.

Next, except for experience in measurement, frequent use of mental calculations involving proportional reasoning is also suggested to be connected with high performance on computational estimation. The other two good estimators (S3 & S4), who demonstrated very high performance in both tests, do mental calculations of proportions frequently.

Student 3 (S3), who works at a bakery, claimed that he calculates the quantities of the ingredients depending on the number of loaves he wants to bake. As he said during his interview: “For example, at the bakery we put 33g of salt in 15L of water. If you have 16L or 10L you have to calculate how much [salt] you need – and you have to do it with your mind at that moment! Also, 1L of water requires 2kg flour and makes 6 loaves of bread. Depending on the bread I want, I calculate the flour.”

Likewise, student 4 (S4) also does mental calculations in her everyday life. In fact, she had been an immigrant in Germany for many years, before she came back to Greece. As a result, she had used both Drachma (Greek former currency) and Deutsche Mark (German former currency) before the transition to Euros, which is the actual currency for both countries. Thus, S4 used Drachma as a benchmark in order to link the other currencies to it. As she said: “I calculated everything with my mind. I did "drachmaization"… both with Marks and with Euros. My husband found it difficult but I could instantly calculate everything in Drachmas.”

To sum up, dealing with everyday activities that involve mental procedures like calculating proportions mentally or estimating measures was reflected as a strong advantage in the performance of the best estimators. Still, it seems possible that they have a positive attitude when they carry through these meaningful activities.

Unlike the best estimators, S13 (with the worst performance) did not reflect his prior experience with money in the pre-test. S13 is a delivery boy and he has to calculate mentally the change he has to give when he delivers the orders. However, as he confessed during the post-test interview, he does not enjoy mental calculations and he prefers that his boss calculates the change for him, before he drives off. For that reason, it is suggested that a positive attitude is necessary to benefit from cognitively demanding everyday activities.

Finally, the rest eight participants (S5 to S12) did not report any frequent or professional involvement in everyday activities that require calculations or measurements.

**Attitudes towards mathematics**

The mean scores of students’ attitudes toward mathematics are presented in Table 7. As shown, most students feel confident, like learning and give high value to
mathematics in general (mean scores > 1). A clear correlation between these attitudes and computational estimation skill cannot be inferred.

**Table 7:** Mean scores and standard deviations in students’ responses concerning their attitudes toward mathematics (confident, like learning and value)

<table>
<thead>
<tr>
<th>Student</th>
<th>Confident Mean (min= –2. max= 2)</th>
<th>S.D.</th>
<th>Like Learning Mean (min= –2. max= 2)</th>
<th>S.D.</th>
<th>Value Mean (min= –2. max= 2)</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2.00</td>
<td>0.00</td>
<td>1.75</td>
<td>0.50</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>S2</td>
<td>2.00</td>
<td>0.00</td>
<td>1.75</td>
<td>0.50</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>S3</td>
<td>1.00</td>
<td>1.41</td>
<td>1.75</td>
<td>0.50</td>
<td>1.75</td>
<td>0.50</td>
</tr>
<tr>
<td>S4</td>
<td>1.00</td>
<td>1.41</td>
<td>1.25</td>
<td>1.50</td>
<td>1.75</td>
<td>0.50</td>
</tr>
<tr>
<td>S5</td>
<td>2.00</td>
<td>0.00</td>
<td>2.00</td>
<td>0.00</td>
<td>1.75</td>
<td>0.50</td>
</tr>
<tr>
<td>S6</td>
<td>0.25</td>
<td>1.50</td>
<td>1.00</td>
<td>2.00</td>
<td>-0.25</td>
<td>2.06</td>
</tr>
<tr>
<td>S7</td>
<td>1.00</td>
<td>1.41</td>
<td>2.00</td>
<td>0.00</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>S8</td>
<td>1.75</td>
<td>0.50</td>
<td>2.00</td>
<td>0.00</td>
<td>1.75</td>
<td>0.50</td>
</tr>
<tr>
<td>S9</td>
<td>-0.25</td>
<td>1.50</td>
<td>1.75</td>
<td>0.50</td>
<td>1.25</td>
<td>1.50</td>
</tr>
<tr>
<td>S10</td>
<td>0.50</td>
<td>1.00</td>
<td>1.75</td>
<td>0.50</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>S11</td>
<td>1.25</td>
<td>0.50</td>
<td>1.50</td>
<td>0.58</td>
<td>1.75</td>
<td>0.50</td>
</tr>
<tr>
<td>S12</td>
<td>2.00</td>
<td>0.00</td>
<td>2.00</td>
<td>0.00</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>S13</td>
<td>1.50</td>
<td>0.58</td>
<td>2.00</td>
<td>0.00</td>
<td>0.75</td>
<td>1.89</td>
</tr>
<tr>
<td>S14</td>
<td>-0.75</td>
<td>1.89</td>
<td>0.50</td>
<td>1.73</td>
<td>-0.75</td>
<td>1.89</td>
</tr>
<tr>
<td>S15</td>
<td>0.25</td>
<td>1.50</td>
<td>1.75</td>
<td>0.50</td>
<td>1.25</td>
<td>1.50</td>
</tr>
<tr>
<td>Mean (N=15)</td>
<td>1.03</td>
<td>1.65</td>
<td>1.27</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is evident, however, that the participants with difficulties in mathematics (S14 & S15), who underperformed, have low confidence in mathematics (-0.75 and 0.25 respectively). It can be argued that it is about a correlation between low confidence and low performance and not an inferential relation, because the cause of their behaviour is probably much more complex. For instance, they may suffer from mathematical anxiety or they could have learning difficulties.

**4.4. Maths teacher’s beliefs about computational estimation**

The classroom maths teacher was interviewed after she implemented the teaching experiment. She said that she was unaware of the concept of computational estimation before her professional learning intervention which was conducted during this study. The only relevant knowledge she had was the rounding strategy, which is instructed in junior high school.

However, it is impressive that she could discriminate rounding from computational estimation after the teaching intervention. It can be assumed that she has a good conceptual understanding of computational estimation due to the experience she gained in this study. First, she understands that estimation reflects
individual approaches. As she said: “in rounding I did not let students free... when I asked them to "round off a number" I told them to which position they should do it. [...] in estimation they can choose [the position].” Second, she claimed that the fairest direction to round off “depends on the problem”. Third, she mentioned the “different procedures” with which estimation problems could be solved implying that rounding is not the only estimation strategy. For instance, she said that “sometimes we only take the first digit under consideration” having the front-end strategy in mind.

Although the maths teacher displayed conceptual understanding of estimation in her interview, she did not demonstrate the expected procedural knowledge in the classroom. Lack of such knowledge of computational estimation strategies can hardly be called into question. Thereby, it is suggested that she has a conservative stance about estimation. Actually, she avoided instructing various computational estimation strategies during the experimental teaching, whilst she overemphasized the rounding off procedure. Therefore, this emphasis explains the significant increase of rounding strategy in post-tests.

The mathematician’s behavior may be attributed to an implicit belief that rounding is the only mathematically correct procedure to estimate, since it is based on rule. Many educators equate computational estimation with rounding, as Alajmi (2009) found. Additionally, as has been noted, estimation can be considered inferior to exact calculation by estimators who have a vague notion of its nature and purpose (Morgan, 1988, as cited in Sowder, 1992, p. 375). Maybe the maths teacher does not fully understand the meaning of estimation.

The truth is that the maths teacher was pretty confident about her content knowledge, because she believes that instrumental strategies are known to mathematicians. It is that she had not taught estimations before and she “only needed a direction” as she said. Overall, she referred to her teaching intervention as a “very good” experience. Furthermore, she characterized the concept of computational estimation as “easy” and “useful” topic, especially for those who are not fluent in the basic operations yet. Also, she found the context of the word problems intriguing and she commented their link with everyday life as something positive.

Although she believes that students’ performance was improved due to their long encounter with estimation (the 7-hour intervention allowed a deep understanding), she commented that she devoted too much time. Namely, she made clear that she felt stretched to teach other mathematical topics too. Maybe this is why her comments were moderate and she was not enthusiastic during the interview. Finally, there was not any spontaneous disclosure of intention to teach estimation to her future students. Thus, it is doubtful that she would engage estimation in her teaching practices.

5. DISCUSSION

The results of the present study documented adults’ computational estimation skills. However, some limitation of the research cannot be ignored. The sample was small,
only a class of 15 adult learners participated. In addition, the tests for the interviews and the materials used for the teaching sessions were adjusted to the specific participants and to the original learning ecology. Due to the small sample and to the nature of the present investigative study, it was not possible to conduct further statistical data analysis that could provide a bit more generalizable results. Besides, the use of typical quantitative assessment tests and methods was avoided, because the methodological choices served the qualitative objectives of the current study. This is why many aspects about the teaching and learning of estimation in Second Chance Schools were revealed.

Regarding participants’ estimation skills, most responses in pre-test (58.1% of total answers) derived from mental procedures, like computational estimation strategies or mental calculations. Rounding was the most frequent estimation strategy, a result that is in accordance with other researches with various samples (Hanson & Hogan, 2000. Tsao & Pan, 2013. Boz & Bulut, 2012. Alajmi, 2009). However, although adults can estimate, there is still much room for improvement.

The teaching experiment conducted in this study investigated some factors that can be crucial for estimation. First, prior knowledge and experience seems to be fundamental for estimation skill. On the one hand, the best estimators had a strong cognitive background deriving from their everyday activities, which involved mental procedures like measuring or calculation proportions. Their everyday experience was reflected in their estimation performance. Lemonidis (2013) suggests that craftsmen like builders are fluent in computational estimations, because they are good at measurements. On the other hand, students with difficulties in maths did not even demonstrate a slight improvement in their rounding skill. This means that they even lack the knowledge of basic procedures.

Next, participants committed most errors in multiplicative reasoning estimation tasks and had a better performance in estimation involving addition and subtraction. Facility with additive estimation problems is also reported by other researchers (Tsao, 2013. Bana & Dolma, 2004. Hanson & Hogan, 2000). Thus, teachers need to evaluate students’ skills in mathematics and help them enhance their number sense by promoting a deeper understanding of numbers and operations. Especially for adults, teachers can use their everyday activities with money as a source of emotional engagement in order to motivate them to improve and practice their estimation skills.

In accordance with the results above, Star et al. (2009) also exhibited the role of prior knowledge in the development of estimation strategy flexibility. They found that students who were fluent estimators at pre-test used to increase their accuracy in estimation, while less fluent students adopted strategies that were easier to implement. In another study, Star & Rittle-Johnson (2009) showed that students who already knew some estimation strategies gained a deeper conceptual understanding of estimation after getting taught some lessons about estimation.

Afterwards, attitudes towards estimation are also important. Participants had a positive attitude towards mathematics in general, except for the two women (S14 &
S15) with the difficulties in maths who were not confident at all. However, their topic-specific attitudes were not investigated. Maybe some students, like S13, did not enjoy mental calculations in particular or did not consider approximate outcomes mathematically correct. Even the maths teacher avoided teaching a mixture of estimation strategies and insisted on rule-based rounding, although she recognized the usefulness of estimation for weak students and she seemed to understand the concept of estimation. Neutral attitudes like this were also reported by Tsao (2013) who explained such elementary teachers’ stances as a result of limited school experience in estimation. This is why estimation should be included in teacher training programmes too.

How can maths educators improve such negative or neutral attitudes towards estimation? By creating positive experiences for both students and teachers. Professional learning interventions have been designed in order to develop teacher’s PCK about computational estimation and to enhance students’ performance in estimation (Mildenhall & Hackling, 2012. Mildenhall et al, 2009). Likewise, in this study, the maths teacher was involved in a professional learning intervention and implemented 7 teaching sessions about estimation. Consequently, the introduction of this topic broadened her teaching practices and was also beneficial for adult learners. Particularly, participants’ mental strategies covered 58.1% of total answers in pre-test and 82.1% in post-test. Their improved performance was also accompanied by a decrease of errors as well.

For all the reasons mentioned before, it is strongly suggested that computational estimation is incorporated into Second Chance Schools and into adult numeracy teaching practices in general. Adult learners will improve their performance and their strategies in estimation. In addition, they are expected to be motivated by contextual estimation problems, because they regard their daily activities. Finally, maths teachers are advised to make sure that all students understand the meaning of estimation and improve their procedural knowledge by enhancing their number sense. In this fashion, even the students who are at disadvantage are going to be helped.

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**BRIEF BIOGRAPHIES**

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