Ἔχεις μοι ἐπιμεῖν, ὦ Σώκρατε, ἄρα διδακτὸν ἡ ἀρετή; ἢ οὐ διδακτὸν ἀλλ' ἀσκητὸν; ἢ οὔτε ἀσκητὸν οὔτε μαθητὸν, ἀλλὰ φύσει παραγίγνεται τὸ ἄνθρωπος ἢ ἄλλῳ τινὶ τρόπῳ.
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The scope of the MEJER is broad, both in terms of topics covered and disciplinary perspective, since the journal attempts to make connections between fields, theories, research methods, and scholarly discourses, and welcomes contributions on humanities, social sciences and sciences related to educational issues. It publishes original empirical and theoretical papers as well as reviews. Topical collections of articles appropriate to MEJER regularly appear as special issues (thematic issues).

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EDITOR'S INTRODUCTORY NOTE

INTRODUCTION TO SPECIAL ISSUE

Behaviour of students, teachers and future teachers in mental calculation and estimation

We are happy to present the first Special Issue of our new journal “MENON: Journal for Educational Research” which was introduced in 2012. Research in Mathematics Education is a significant area of educational research, which is included in the topics of this journal.

“Behaviour of students, teachers and future teachers in mental calculation and estimation” has been chosen as the subject for this special issue on the ground of a number of reasons which are presented below.

Over the past decades, many studies have been conducted in the field of mental calculations and estimation and more precisely in relation to the definition of these concepts, the identification of the strategies used by various age groups, the relationship with other concepts, such as number sense, the procedural and conceptual understanding among others.

Many educational systems have updated the teaching of numbers and operations in mathematics, incorporating mental calculations and estimations in their elementary and middle education curricula.

Nowadays, it is considered timely to conduct research in the implementation of the teaching of mental calculation and estimation with whole and rational numbers as well as the recording of students' behaviour and the training of pre-service and in-service teachers in these concepts.

During the last decade, researches on mental computation and estimation with rational numbers has been conducted in the Laboratory of “Nature and Life Mathematic” at the University of Western Macedonia, some of which are presented in this issue.

Most of the papers included in this issue, refer to mental calculations and estimations with rational numbers, a topic that is not very common in the literature and covers a wide age range including elementary school students, adults, as well as pre- and in-service teachers. The researches are presented according to the age range of the participants.

- In their study Greet, Bert, Torbeyns, Ghesquière and Verschaffel distinguish between two types of strategies for subtraction: (1) direct subtraction, and (2) subtraction by addition, and provide an overview of the results of 5 related studies using non-verbal methods to investigate the flexible use of these strategies in both single- and multi-digit subtraction. Adults, students and
elementary school students with mathematical learning disabilities have participated in this research.

- **Anestakis and Lemonidis** in their study, investigate the computational estimation ability of adult learners and implement a teaching intervention about computational estimation in a Junior High School for Adults. They suggest incorporating computational estimation into Second Chance Schools and into adult numeracy teaching practices in general.

- The two papers of Lemonidis, Nolka, Nikolantonakis and Lemonidis, Kaiafa examine the behaviour of 5th and 6th grade students in computational estimation and in mental calculations with rational numbers, respectively. In these studies, the relation between students’ performance in computational estimation and mental calculations with rational number and problem solving ability are also examined.

Four studies on this issue, refer to the prospective elementary teachers’ behaviour in mental calculation and estimation.

- Anestakis and Desli examined 113 prospective primary school teachers’ views of computational estimation and its teaching in primary school. Results revealed that the majority of prospective teachers identified the importance of computational estimation for both daily life and school.

- In their research Kourkoulos and Chalepaki interviewed and examined through a test 69 pre-service teachers aiming to investigate the factors that contribute to their computational estimation ability. They found five factors that contribute to computational estimation, such as the mathematical background and the attitude towards mathematics.

- Lemonidis, Tsakiridou, Panou and Griva used interviews to examine the knowledge and the strategy use of 50 pre-service teachers in multiplication tables and their mental flexibility in two-digit multiplications by using the method choice / no-choice by Lemaire & Siegler, (1995). The results showed that prospective teachers are not flexible in two-digit multiplications and their flexibility to mentally calculate two-digit multiplications is associated with their knowledge in prep and their prep response time.

- Koleza and Koleli have used a test to study the mental computations and estimation strategies of 87 pre-service teachers. The data revealed that the prospective teachers’ number sense concerning rational numbers, basic concepts of the decimal system and elementary numerical properties was very weak.

- Lemonidis, Mouratoglou and Pnevmatikos studied 80 in-service teachers’ performance and strategies in computational estimation and individual
differences concerning their age and work experience, their attitude towards mathematics and their prior performance in mathematics during high school years.

- The last paper of Lemonidis, Kermeli and Palaigeorgiou propose a teaching intervention to sixth grade students in order to promote understanding and enrich their conceptual strategy repertoire to carry out mental calculations with rational numbers. At the same time, three teachers’ attitudes towards teaching mental computation with rational numbers, were examined.

Finally, I would like to thank all the researchers from Belgium and Greece who contributed with their papers in this thematic issue, the colleagues from the laboratory of "Nature and Life Mathematics", the reviewers of the papers and Elias Indos for the organizational and technical support in the journal.

The Editor of the first Special Issue of “MENON: Journal for Educational Research”

Charalampos Lemonidis
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FACTORS CONTRIBUTING TO COMPUTATIONAL ESTIMATION ABILITY OF PRESERVICE PRIMARY SCHOOL TEACHERS

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ABSTRACT
This study focuses on the investigation of the factors that are related and contribute to computational estimation ability. We interviewed 69 students of the Department of Primary Education of the University of Crete. Moreover, these students had filled in a test. According to the analysis of the results, the factors that contribute to success at computational estimation are:

- good mathematical background and mainly good performance at exact mental computation and proportion problems,
- preference to mathematics at school,
- positive self-concept of computational estimation ability,
- positive self-concept of acquiring exact mental computation ability from the first grades of primary school,
- positive self-concept of memory ability and particularly numerical data memory ability.

Keywords: computational estimation, preservice teachers, mathematical background, affective factors.

1. INTRODUCTION
Computational estimation can play a vital role in mathematical education because it contributes to better understanding, learning and applying the algorithms (Kourkoulos and Tzanakis, 2000), as well as better understanding of numbers and their properties (Sowder, 1992). Montague and Garderer (2003) refer that computational estimation ability is associated with the acquisition of number sense

1 McIntosh et.al (1997) describe number sense as “a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations” (p. 3).
while Mildenhall (2011) believes that computational estimation is an integral component of number sense.

Computational estimation can be mentally, before applying the algorithm of an operation. Moreover its results can be used during the application of the algorithm in order to check the correctness of the process that is followed. Besides, they can be used after applying the algorithm in order to check the correctness of the produced result (Kourkoulos and Tzanakis, 2000). In case of the existence of an error, the application of self-correction activities is the next step. In case that a student applies such activities, he tries to find the error and correct it. However, self-correction is a basic metacognitive skill that students must develop through systematical teaching, because the acquisition of metacognitive skills: a) enables students for self-regulated learning and b) cultivates students’ positive attitude to knowledge (Matsaggouras, 2000).

Additionally, computational estimation can be used for checking the results that are produced by a calculator or a computer (Kourkoulos and Tzanakis, 2000), because there is often wrong insert of data that leads to wrong results.

Moreover, computational estimation can substitute the algorithm itself in everyday life situations where there is not the necessity of the exact result but estimation is enough. In these cases, the estimation of the result has the advantage that it usually produces faster results than the use of paper and pencil or even the calculator, when there is the appropriate exercise (Kourkoulos and Tzanakis, 2000). It is significant that it does not demand material support and consequently it can be easily applied in cases that we don’t have paper and pencil or calculator, such as when we want to estimate the cost of various products in a super market and make the decision if the money we have are enough to buy them (Lemonidis, 2013).


2. FACTORS THAT ARE RELATED TO COMPUTATIONAL ESTIMATION

The existing research on the factors related to computational estimation ability is limited; nevertheless this research identifies some such factors that can be classified in two categories: a) cognitive factors and b) affective factors. Findings of the existing research concerning these two categories are discussed below (sections 2.1 & 2.2).

2.1 Relation between computational estimation and other cognitive factors

The cognitive factors related to computational estimation ability that are mentioned in existing research studies can be classified in two groups: (i) specific cognitive factors and (ii) general cognitive factors, and more specifically general mathematical ability.
i) With regard to the first group:

- Ability to work with powers of ten is one factor that is related to computational estimation ability (Sowder and Wheeler, 1989, Rubinstein, 1985). Kourkoulos and Tzanakis (2000) mention that a prerequisite for the application of the estimation and checking criteria that rely on mental approximate computation is ability to compute mentally the results of operations with numbers that are powers of ten and with numbers of the format $nx10^m$ (where $m \in \mathbb{Z}$ and $n$ is of one digit, or at a more advanced level $n$ is of two digits), especially multiplications and divisions (e.g. $700 \times 8000 = \cdot$, $0.02 \times 0.003 = \cdot$). However, only Rubinstein presents experimental data according to which operating with tens contributed the most to the prediction of estimation performance according to regression analysis.2


- Ability to compare numbers by size is another factor that is related to computational estimation ability (Sowder and Wheeler, 1989, Kourkoulos and Tzanakis, 2000, Rubinstein, 1985).

- Other concepts and skills mentioned to be related to computational estimation ability are: a) knowledge of basic facts, b) knowledge of properties of operations and their appropriate use, c) recognition that modifying numbers can change outcome of computation (Sowder and Wheeler, 1989) and d) problem difficulty level (Dowker, 1997). However, research on examining the relation between these factors and computational estimation is very limited.

Regarding all the above studies we remark that although they indicate some specific cognitive factors which are related to computational estimation ability, most of them don’t present any experimental data and rely on theoretical analysis. Besides, there is no study (except for Rubinstein, 1985) that examines the correlation between computational estimation ability to other factors according to statistical analysis.

- Moreover, there are other specific factors that may be related to computational estimation ability but this relation hasn’t been investigated so far by researchers. Algorithmic performance of arithmetic operations could be one related factor. One reason that could justify this relation is that computational estimation can be used before the performance of an arithmetic operation and its result can be used in order to check the correctness of the result that is produced after the algorithmic performance of the operation. Besides, applying computational estimation strategies involves deep understanding of arithmetic operations.

2 Rubinstein’s experimental data concern eighth graders. According to regression analysis operating with tens accounted for 42% of the variance in the estimation test score. The other mathematical skills that contributed to the prediction of estimation performance is making comparisons and getting to know the problem. These three factors altogether accounted for 46% of the variance in the estimation test score.
Ability to solve proportional problems could be another factor that is related to computational estimation ability. Its relation to computational estimation (of multiplications and divisions) is possibly due to the fact that they both belong to multiplicative structures. Besides, understanding proportion problems may contribute to computational estimation ability. For example, during the estimation of the result of a multiplication by rounding, compensation can take place in order to find a better approximation where proportional reasoning is being used: it is counted what about percentage has been cut by the one or the other one factor of the operation or has been given to them so as the result to be compensated.

Finally, ability to solve additive problems could be another factor related to computational estimation ability. This relation could be justified by the fact that compensation, which takes place in order to achieve a better approximation, involves various estimation strategies which involve posing and solving additive problems. Besides, during solving an additive problem there is the need to estimate the answer of the problem in order to check the reasonability of it.

**ii) With regard to the second group (general cognitive factors):**

Hogan and Brezinsky (2003) concluded that computational estimation may be subsumed under more general, well-established mathematical abilities, specifically a combination of number facility and quantitative reasoning which they call general mathematical ability. Computational estimation is not a unique ability but it is a part of this general mathematical ability. Levine (1982) found a positive correlation between math ability and estimation skill. Cilingir and Turnuklu (2009) found that students with high mathematical level and high mathematics grades are better at computational estimation. However, Gliner (1991) tried to determine the factors that contribute to computational estimation ability of preservice elementary teachers and found that average mathematics grades were negatively correlated to computational estimation performance. The fact that Gliner’s findings are not in line with other research findings points out that the relationship between computational estimation ability and general mathematical ability needs clarification. In fact, on this issue basic questions still remain without sufficient answer, such as “which are the adequate indicators of general mathematical ability which can be used so as to allow the efficient investigation of the aforementioned relationship?”

Concerning the existing research on cognitive factors related to computational estimation ability, overall we can remark that there are factors not yet or very little considered; moreover regarding factors that have received some notable attention there is an important lack of empirical investigation, in particular concerning their strength of association, as well as, their combined relation to computational estimation ability. Existing research informs us that the subject is a complex one and important research work remains to be done for elaborating a more complete understanding concerning which cognitive factors are related to computational estimation ability and what the strength and the way of their association are.
2.2. Affective factors related to computational estimation

There are few studies that investigate the affective factors which are related to computational estimation ability. In these studies, five basic affective factors have aroused that seem to be related to estimation and characterize those who have computational estimation ability: a) positive attitude to computational estimation process, b) positive self-concept of computational estimation ability, c) positive self-concept of mathematical ability d) preference to mathematics and e) tolerance for error.

- Positive attitude to computational estimation is a factor that is related to computational estimation ability and characterizes good estimators according to researchers’ positions (Sowder, 1992, Sowder and Wheeler, 1989) and findings (Bestgen et al., 1982, Reys et.al, 1982, LeFevre, 1993). However, Boz and Bulut (2012) found that seventh grade students’ perceptions in the practicality of computational estimation are not related to computational estimation ability, as some good estimators believe that estimation has practicality in daily life but not in mathematics. In a study with English 12-to 14-year-olds, Morgan (1988) as well found that most of the children she interviewed did not have a clear conception of the purpose or the nature of estimation.

- Another factor that is related to computational estimation ability is positive self-concept of estimation ability (Reys et.al, 1982, LeFevre, 1993). However, in a study of preservice elementary teachers, Gliner (1991) found that there is no correlation between computational estimation and self-perception of their own estimation ability.

- Positive self-concept of mathematical ability is a characteristic of good estimators (LeFevre, 1993, Sowder, 1989, Gliner, 1991). However, Boz and Bulut (2012) found that confidence in ability to do mathematics is a component of good estimators but not a distinctive factor.

- Preference to mathematics may be another factor related to computational estimation ability. However, this relation is very little investigated by existing research. Gliner (1991) examined this relation and found a positive correlation between these two factors.

- Tolerance of error is another factor that is related to computational estimation ability. Reys et al. (1980) found that tolerance for error is a characteristic of good estimators because it depends on the understanding of an estimate. Wyatt (1986) supports that good estimators are much less concerned about precision than poor estimators. Boz and Bulut (2012) found that good estimators had high tolerance for error.

Regarding these studies we can remark that relation between computational estimation and affective components has received very little attention and there are few quantitative experimental data regarding this relation. Moreover, although some
affective factors related to computational estimation have been determined, there isn’t a consensus of findings about their relation to computational estimation. Besides, other factors that are possibly related to computational estimation such as self-concept of mental computation ability, self-concept of memory ability, ambitions etc. have not been investigated.

Taking into account the above ascertentions, we designed and implemented an empirical investigation for contributing to the research concerning the factors that are related and contribute to computational estimation ability.

Our empirical investigation, which concerns preservice primary school teachers, aims to provide elements of answer to the following research questions:

- Which cognitive and affective factors are related to computational estimation ability?
- Which is the correlation between these factors and computational estimation ability?
- How groups of such factors are related to computational estimation ability?

3. METHODOLOGY

3.1 Participants

In order to achieve the purpose of the investigation and provide elements of answers to the research questions, 69 preservice elementary teachers, students of the 2nd semester of the Primary Education Department of the University of Crete (Greece), were interviewed. These students were selected from 244 students who had filled in a test during the attendance of a course of mathematical content. The students who participated to the interview were selected randomly, according to their performance at the test:

- a) they had good performance at computational estimation (14 students)
- b) they had moderate performance at computational estimation (20 students)
- c) they had low performance at computational estimation (35 students)

3.2 Materials and procedure

3.2.1 Test

The test that was developed consisted of items concerning both computational estimation and other mathematical areas, from which we selected typical items that represent them. The items of other mathematical areas were included in the test in

---

3 Good performance was considered the success at least at 14 of the 18 items of computational estimation and at least at 3 of the 6 items of estimating multiplications of integrals and at least at 2 of the 4 items of the rest areas of computational estimation (multiplications of decimals, divisions of integrals and divisions of decimals. Moderate performance was considered the success at 9-13 items of computational estimation. Law performance was considered the success at 0-8 items of computational estimation. We selected students of each category randomly at equal percentage of the corresponding totals (we selected randomly 14 of 49 students who had good performance, 20 of 71 students who had moderate performance and 35 of 124 students who had low performance).
order to investigate whether there is any relation between computational estimation and other mathematical skills. We selected items of other mathematical areas for which we have reasons to believe that are related to computational estimation (see section 2.1).

Specifically the test consisted of:

a) computational estimation
b) exact mental computation with numbers that are power of 10 and with numbers of the format \(nx10^m\), where \(n \in N^*, 1<n<100\) and \(m \in Z\)
c) algorithmic performance of arithmetic operations
d) additive and multiplicative structure problems

The description of the items of the test is the following one:

**a) Computational estimation**

i) Find mentally between which numbers the results of the following operations are and put a cross in the appropriate small square:

<table>
<thead>
<tr>
<th>Multiplications</th>
<th>Divisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrals</td>
<td>57•83</td>
</tr>
<tr>
<td>Decimals</td>
<td>9,24•0,27</td>
</tr>
</tbody>
</table>

A solved example was presented to the participants:

13 • 11 10 < □ < 100 < ⊕ < 1.000 < □ < 10.000 < □ < 100.000 < □ < 1.000.000 < □ < 10.000.000 < □ < 100.000.000 < □ < 1.000.000.000

ii) Compute mentally how much about the result of the following operations is (e.g. 23•11 about: 200):

<table>
<thead>
<tr>
<th>Multiplications</th>
<th>Divisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrals</td>
<td>62•73</td>
</tr>
<tr>
<td>Decimals</td>
<td>8,32•0,26</td>
</tr>
</tbody>
</table>

**b) Exact mental computation with numbers that are powers of 10 and with numbers of the format \(nx10^m\), where \(n \in N^*, 1<n<100\) and \(m \in Z\)**

<table>
<thead>
<tr>
<th>Multiplications</th>
<th>Divisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>with positive powers of ten</td>
<td>with numbers of the format (nx10^m), where (n \in N^*, 1&lt;n&lt;100) and (m \in Z)</td>
</tr>
<tr>
<td>3,85•10000</td>
<td>600•700</td>
</tr>
<tr>
<td>0,0467•10000</td>
<td>90•80000</td>
</tr>
<tr>
<td>with negative powers of ten</td>
<td>with numbers of the format (nx10^m), where (n \in N^*, 1&lt;n&lt;100) and (m \in Z)</td>
</tr>
</tbody>
</table>
3.2.1. Algorithmic performance of arithmetic operations

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4623</td>
<td>0,01</td>
<td>0,02</td>
<td>0,001</td>
</tr>
<tr>
<td>25,4</td>
<td>0,001</td>
<td>0,06</td>
<td>0,07</td>
</tr>
<tr>
<td>3,57</td>
<td>0,001</td>
<td>2,4</td>
<td>0,03</td>
</tr>
<tr>
<td>25,4</td>
<td>0,0001</td>
<td>0,06</td>
<td>0,07</td>
</tr>
<tr>
<td>67</td>
<td>0,01</td>
<td>0,02</td>
<td>1:7</td>
</tr>
<tr>
<td>0,004</td>
<td>2,4</td>
<td>0,03</td>
<td></td>
</tr>
<tr>
<td>0,007</td>
<td>0,02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2.2. Additive and multiplicative structure problems

i) Proportion problems
- In a factory, a machine produces chocolate. The machine produces 325 kilos of chocolate at 52 minutes. How long does it take the machine to produce 260 kilos of chocolate?
- In a factory producing ice-cream, 5 same machines worked for 18 minutes each one and produced 720 kilos of ice-cream all together. If 14 machines work for 36 minutes each one, how much ice-cream will they produce all together?

ii) Additive structure problems
- John played two rounds of marbles. In the first round he won 16 marbles. At the end of the game he has won 5 marbles. What happened during the second round? Did he win or did he lose and how many marbles?
- George played two rounds of marbles. In the first round he lost 9 marbles. At the end of the game he has won 5 marbles. What happened during the second round? Did he win or did he lose and how many marbles?

Students had about 85 minutes to fill in the test.

3.2.2. Interview

The questions that the interview contained were put in order to let us determine the factors that are related to computational estimation ability. The questions can be classified in the following categories:
- Social characteristics
- Characteristics concerning participants’ knowledge background
- Attitudes and preferences
- Characteristics concerning participants’ self-concept
- Ambitions

More specifically the questions that the interview contained are:

Social characteristics
- Gender
- Parents’ knowledge level
- Place of origin

Participants’ knowledge level (background)
- Direction at secondary school
- Mathematics grades at secondary school (10th, 11th and 12th grade)
- Physics grades at secondary school (10th, 11th and 12th grade)
- Grade point average at 11th grade
- Grade point average at 12th grade
- Mathematics grade at university
- Physics grade at university
- Knowledge of foreign languages and at what level

**Self-concept**

- Mathematical ability
  
  *Do you believe that you are good at mathematics?*
  *At which courses do you believe that you are good?*

- Exact mental computation ability and age at which this ability was acquired
  
  *Can you easily compute mentally (e.g. 8•36  12•15)?*
  *If you can, from what age about have you been able to compute mentally?*

- Computational estimation ability
  
  *Do you believe that you are good at computational estimation (e.g. how much is about 289•324);*

- Memory ability
  
  *Have you got good memory?*
  *Do you remember many telephone numbers by heart?*

**Questions concerning participants’ attitudes and preferences**

- Attitude to the meaning of computational estimation and reasoning this attitude
  
  *Do you believe that computational estimation is important?*
  
  - Very important ☐
  - Little important ☐
  - Quite important ☐
  - Not important ☐

  *If it is, why is it important?*

- Attitude to checking operations results and reasoning this attitude

- Courses that they liked more at school

- Courses they like more at university

- Liking mathematics
  
  *Do you like Mathematics?*

**Questions concerning participants’ ambitions**

*What are you going to do after graduating the university?*

*The interview was personal and lasted about 30-35 minutes.*

---

4 Self-concept is the convictions and attitudes that an individual forms about himself, which involve various self-evaluations and behavioral tendencies (Burns, 1982).

5 Preference belongs to the field of attitudes as it involves the meaning of liking – sentimental dimension of attitude – and the meaning of behavioral tendency or intention – behavioral dimension of attitude (see Al-Khaldi and Al-Jabri, 1998).
4. RESULTS

Correlation tests were used in order to determine the factors that are related to success at computational estimation and the strength of this correlation. Besides, stepwise regression analysis was used in order to determine the factors that possibly contribute and can predict success at computational estimation. This method lets us predict the dependant variable -success at computational estimation - based on a combination of independent variables -factors that are related significantly to the dependent variable (Dafermos, 2005, see also footnote 6). This analysis informs us about the way in which the related factors altogether function and permit prediction of success at computational estimation.

The score at computational estimation is the dependant variable (range 0-18) and all the variables concerning the test and the interview that are significantly correlated to the dependant variable (according to Pearson r) are the independent variables. The variables that entered into the model are in table 1:

Table 1: Stepwise multiple regression analysis to predict computational estimation score

<table>
<thead>
<tr>
<th>Variables</th>
<th>R</th>
<th>R²</th>
<th>Adjusted R²</th>
<th>R² change and F change</th>
<th>Durbin-Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R² change</td>
<td>F change</td>
</tr>
<tr>
<td>1</td>
<td>0,599</td>
<td>0,359</td>
<td>0,354</td>
<td>0,359</td>
<td>37,54</td>
</tr>
<tr>
<td>2</td>
<td>0,717</td>
<td>0,514</td>
<td>0,499</td>
<td>0,155</td>
<td>21,075</td>
</tr>
<tr>
<td>3</td>
<td>0,793</td>
<td>0,63</td>
<td>0,612</td>
<td>0,115</td>
<td>20,237</td>
</tr>
<tr>
<td>4</td>
<td>0,824</td>
<td>0,679</td>
<td>0,659</td>
<td>0,05</td>
<td>9,913</td>
</tr>
<tr>
<td>5</td>
<td>0,851</td>
<td>0,724</td>
<td>0,702</td>
<td>0,045</td>
<td>10,224</td>
</tr>
<tr>
<td>6</td>
<td>0,866</td>
<td>0,749</td>
<td>0,725</td>
<td>0,025</td>
<td>6,226</td>
</tr>
</tbody>
</table>

1. Exact mental computation
2. They mention preference to the course of mathematics at school
3. Self-concept of computational estimation ability
4. Proportion problems
5. Self-concept of mental computation ability having been acquired from the first grades of primary school
6. Self-concept of telephone numbers memory

On each step an independent variable that increases R-Squared the most enters into the model, conditioning that this increase is statistically significant at significant level less than 0,05. Then, the next variable that increases significantly R-Squared the most enters. Then, the inserted variables are checked to see if one of them satisfies the removal criterion. That is the variable that increases R-Squared the least is removed, conditioning that this increase is non-significant at significant level 0,01. Then the variable that increases R-Squared the most enters into the model, conditioning that this increase is statistically significant at significant level less than 0,05 and all the inserted variables are checked to see if they satisfy the removal criterion, e.t.c. Therefore, a variable enters into the model if it is recognized to be a significant predicting factor and it is removed if once it stops to be a significant predicting factor. Given that the significant level is less for the entrance of a variable than the significance level for the removal of a variable, the procedure does not get into an infinite loop (Dafermos, 2005).

The success at each item of computational estimation accounts for one point (see the description of the test).

The statistical variables (1) “Exact mental computation” and (4) “Proportion problems” are the students’ scores at the corresponding areas of the test (see the description of the test). The success at each item of an area...
In table 2 there is the correlation between the variables which entered into the model and the dependent variable:

**Table 2:** Correlation between the variables which entered into the stepwise multiple regression model and the computational estimation score

<table>
<thead>
<tr>
<th>Variables</th>
<th>Computational estimation score</th>
<th>Pearson r</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact mental computation</td>
<td>0,599</td>
<td>&lt;0,001</td>
<td></td>
</tr>
<tr>
<td>They mention preference to the course of mathematics at school</td>
<td>0,565</td>
<td>&lt;0,001</td>
<td></td>
</tr>
<tr>
<td>Self-concept of computational estimation ability</td>
<td>0,517</td>
<td>&lt;0,001</td>
<td></td>
</tr>
<tr>
<td>Proportion problems</td>
<td>0,587</td>
<td>&lt;0,001</td>
<td></td>
</tr>
<tr>
<td>Self-concept of mental computation ability having been acquired from the first grades of primary school</td>
<td>0,395</td>
<td>0,001</td>
<td></td>
</tr>
<tr>
<td>Self-concept of telephone numbers memory</td>
<td>0,251</td>
<td>0,037</td>
<td></td>
</tr>
</tbody>
</table>

However, we should note that there are variables which don’t enter into the model although they are significantly correlated to the dependent variable. This is owing to the fact that they are significantly correlated to variables that have already entered into the model and the main part of their predictability is in the already inserted variables. Therefore, they cannot enter because there would be the phenomenon of multicollinearity (Dafermos, 2005). These variables\(^9\) are shown in table 3:

**Table 3:** Correlation between the variables that don’t enter into the stepwise multiple regression model and computational estimation score under the control of the variables that have entered into the model (partial correlation)

<table>
<thead>
<tr>
<th>Category of questions</th>
<th>Variables (see footnote 9)</th>
<th>Responses</th>
<th>Correlation to computational estimation score (Pearson r)</th>
<th>Correlation to estimation score under the control of entered variables (Pearson r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social characteristics</td>
<td>Gender</td>
<td>Boy</td>
<td>0,331*</td>
<td>0,119</td>
</tr>
</tbody>
</table>

accounts for one point at the corresponding student’s score. The variables (2), (3), (5), (6) are statistical variables with two values (1, 0). For each variable, the value 1 is attributed to a student if his/her answer is affirmative to the corresponding question; for all other answers the value 0 is attributed to the student.

\(^9\) The statistical variables “Additive problems”, “Subtractions” and “Divisions” are the students’ scores at the corresponding areas of the test (see the description of the test). The success at each item of an area accounts for one point at the corresponding student’s score. All the other statistical variables of this table have two values (1, 0). For each variable, the value 1 is attributed to a student if his/her answer corresponds to what is described in the “response” column of the table; for all the other answers the value 0 is attributed to the student.
<table>
<thead>
<tr>
<th>Category of questions</th>
<th>Variables (see footnote 9)</th>
<th>Responses</th>
<th>Correlation to computational estimation score (Pearson r)</th>
<th>Correlation to estimation score under the control of entered variables (Pearson r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background</td>
<td>Additive problems</td>
<td>Score at each of these areas</td>
<td>0.262*</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>Subtractions</td>
<td></td>
<td>0.285*</td>
<td>-0.147</td>
</tr>
<tr>
<td></td>
<td>Divisions</td>
<td>Sciences¹⁰/ Technological</td>
<td>0.44**</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>Direction at school</td>
<td>≥18,5</td>
<td>0.237*</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>Grade point average at 11th grade</td>
<td>≥18,5</td>
<td>0.285*</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>Grade point average at 12th grade</td>
<td>≥18,5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Math grades at school</td>
<td>19-20</td>
<td>0.488**</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>Physics grades at school</td>
<td>19-20</td>
<td>0.35*</td>
<td>-0.141</td>
</tr>
<tr>
<td>Attitudes and preferences</td>
<td>They mention preference to the course of math at university</td>
<td>Yes</td>
<td>0.495**</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>They mention preference to the course of physics at school</td>
<td>Yes</td>
<td>0.39**</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>They mention preference to the course of physics at university</td>
<td>Yes</td>
<td>0.267*</td>
<td>-0.157</td>
</tr>
<tr>
<td></td>
<td>If they like mathematics</td>
<td>Yes</td>
<td>0.543**</td>
<td>0.113</td>
</tr>
<tr>
<td></td>
<td>If they believe that computational estimation is important and reasons for that</td>
<td>Yes and they mention more than one reason</td>
<td>0.385**</td>
<td>0.188</td>
</tr>
<tr>
<td>Self-concept</td>
<td>They mention that they believe that a course at which they are good is mathematics</td>
<td>Yes</td>
<td>0.567**</td>
<td>0.083</td>
</tr>
</tbody>
</table>

¹⁰ In Greece both the direction of Sciences and the Technological direction involve the course of Mathematics while Theoretical direction does not involve this course.
<table>
<thead>
<tr>
<th>Category of questions</th>
<th>Variables (see footnote 9)</th>
<th>Responses</th>
<th>Correlation to computational estimation score (Pearson r)</th>
<th>Correlation to estimation score under the control of entered variables (Pearson r)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>They mention that they believe that a course at which they are good is physics</td>
<td>Yes</td>
<td>0.382**</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>If they believe that they are good at mathematics</td>
<td>Yes</td>
<td>0.534**</td>
<td>-0.044</td>
</tr>
<tr>
<td></td>
<td>If they believe that they are fluent at mental computation</td>
<td>Yes</td>
<td>0.449**</td>
<td>0.05</td>
</tr>
</tbody>
</table>

*Correlation is statistically significant at significant level less than 0.05.
**Correlation is statistically significant at significant level less than 0.001.

We remark that the correlation between these variables and computational estimation score is statistically significant (column 4). However, the correlation between these variables and computational estimation score isn’t statistically significant any more under the control of the variables that have entered into the regression model (column 5), because of their correlation to these variables. This is the reason why these variables do not enter into the model, as their prediction potential is covered by the variables that have already entered into the model.

The variables that entered into the regression model concern the following categories of characteristics (see graph 1 and table 1):

- Mathematical background – cognitive factors which account for 40.9% of R-Squared (exact mental computation accounts for 35.9% of R-Squared, proportion problems account for 5% of R-Squared).
- Preferences (preference to the course of mathematics at school) which account for 15.5% of R-Squared
- Self-concept (accounts for 18.5% of R-Squared) of:
  - computational estimation ability which accounts for 11.5% of R-Squared
  - exact computation ability having been acquired from the first grades of primary school which accounts for 4.5% of R-Squared
  - memory ability (telephone numbers memory) which accounts for 2.5% of R-Squared.
Graph 1: Factors that compose R-Squared according to stepwise multiple regression analysis

* Variance that is not predicted by factors that have been included in the model.

In the beginning, cognitive factors which concern mathematical background seem to play a vital role for the success at computational estimation and especially exact mental computation (prerequisite for computational estimation by applying rounding strategies) and proportion problems.

Besides, their self-concept of computational estimation ability and early acquisition of exact computation ability seem to be very important. In addition to these, there is evidence for the important role of participants’ positive self-concept of numerical data memory ability.

Also, participants’ preference to mathematics seems to be very significant.

All these factors account for 74.9% of the total variance in computational estimation score, which is a high percentage for the field of Social Sciences.

5. DISCUSSION

Data analysis indicated a number of factors that are related to computational estimation ability (tables 2, 3). Besides, it indicated the strength of the correlation of these factors to success at computational estimation (tables 2, 3). In addition, some of them can be predictors for success at computational estimation (table 1, graph 1). Therefore, the analysis determined the combined way in which some factors interact and predict success at computational estimation. So, this study, providing elements to the research questions, lights aspects of computational estimation ability that had not been investigated. The factors that seem to be very important for computational estimation ability are the following ones:

11 The graph shows the R-Squared change that the factors which enter into the model cause, according to table 1.
a) Background

i) Specific cognitive factors of mathematical background

The first cognitive factor that is significantly related and contributes to success at computational estimation is exact mental computation with powers of ten and with numbers of the format $nx10^m$, where $n \in \mathbb{N}$, $1<n<100$ and $m \in \mathbb{Z}$ (table 1, 2) which is prerequisite for computational estimation when using rounding strategies (Kourkoulos and Tzanakis, 2000). This result is consistent with the findings and the positions of other studies (Sowder and Wheeler, 1989, Rubinstein, 1985, Kourkoulos and Tzanakis, 2000) but in the present study this factor is not only significantly correlated to success at computational estimation (table 2) but it is a major factor that contributes to it (table 1).

The second significant cognitive factor is ability to solve proportion problems. The statistical variable that concerns students’ ability to solve proportion problems has important correlation to their computational estimation score (table 2). Moreover, it is a variable that contributes to prediction of success at computational estimation (table 1). These findings are interesting because they point out that there is an important relation between the ability to solve proportion problems and computational estimation ability\textsuperscript{12}. However, this relation hadn’t been investigated before. Nevertheless, our investigation is a first investigation on this issue, and further research is needed to obtain a deeper and more complete understanding of this important relation.

Besides, there is a significant correlation between computational estimation score and scores concerning other mathematical areas and specifically divisions, subtractions and additive problems which they don’t enter into the model because of their relation to other variables that have already entered into the model (table 3). There is no previous research concerning the relations between students’ performances to the above mathematical areas and to computational estimation.

Taking into account the significant relations between the aforementioned factors and success at computational estimation, we can suppose that mathematical background plays a vital role for computational estimation ability.

ii) General background

In the present study we used math grades at school as an indicator of general mathematical ability and we remarked that there is an important correlation between this indicator and computational estimation score (table 3). This result is in line with previous research findings (Levine, 1982, Hogan and Brezinsky, 2003, Cilingir and Turnuklu, 2009) but contrasts the result of Gliner’s study (1991) who found that preservice elementary teachers’ average mathematics grades were negatively correlated to computational estimation performance.

\textsuperscript{12} A possible explanation for the relation between the ability to solve proportion problems and computational estimation is presented in section 2.1.
Moreover we used grade point average at 11th and 12th grade, physics grades at school and direction at school as indicators of general background. All these indicators have a significant correlation to computational estimation score (table 3). However, they don’t enter into the model because their prediction potential is contained in factors that have entered into the model. This information is interesting as these factors and their relation to computation estimation hadn’t been investigated before.

Nevertheless, further research is needed for elaborating a deeper and more complete understanding concerning the way in which these factors are related to computational estimation ability. Moreover, so far research on the subject had not investigated factors of general background, besides general mathematical background; our results indicate that interesting and fruitful research work remains to be done concerning this area.

b) Preferences
Participants’ preference to the course of mathematics at school has an important relation to success at computational estimation (table 2) and it is a major factor that contributes to it (table 1). Besides, liking mathematics, preference to the course of mathematics at university, preference to the course of physics at school and preference to the course of physics at university are factors which are significantly related to success at computational estimation (table 3). These factors don’t enter into the regression model because they are significantly correlated with other variables that have already entered the model. These findings point out that there is an important relation between preference to mathematics and physics and computational estimation ability. This relation has been investigated very little by previous researches. Our findings are in line with the results of Gliner’s study (1991). However, in the present study this relation arouses from more aspects using more indicators and it is more intensive.

Of course, it is possible that the correlation of preference to mathematics to success on computational estimation is owing to the two-sided relation between preference to mathematics and performance at mathematics. Students who like mathematics are interested in doing mathematics and consequently they have possibly better performance. Besides, students who have good performance at mathematics acquire positive attitude to it.

c) Self-concept
Participants’ self-concept seems to play an important role for success at computational estimation. Specifically:

i) Self-concept of computational estimation ability
Self-concept of computational estimation ability seems to be very important for success at computational estimation (tables 1, 2), supporting previous research findings (Reys et.al, 1982, LeFevre, 1993) but contrasting Gliner (1991) who found that there is no correlation between computational estimation and self-
perception of estimation ability. So, according to the present study, it seems that participants’ image of their computational estimation ability complies with their real image.

**ii) Self-concept of exact mental computation ability and the age of acquiring this ability**

Participants’ self-concept of the age of acquiring exact mental computation ability (from the first grades of primary school) is an important factor for predicting success at computational estimation (tables 1, 2) that hadn’t been investigated before. Besides, self-concept of mental computation ability is a factor that has an important correlation to success at computational estimation (table 3) that hadn’t been investigated before too. Therefore, if the image that participants have for themselves with regard to exact mental computation ability and the age of acquiring this ability complies with their real image, there is evidence that early exercise on mental computation and acquisition of this ability may contribute to the development of computational estimation ability.

**iii) Self-concept of memory ability**

Students’ self-concept of memory ability and especially of numerical data memory ability emerges to be an important factor for success at computational estimation (tables 1, 2) that hadn’t been investigated before. The significance of memory on computational estimation ability possibly relies on the fact that estimating the result of an operation by using rounding strategies demands: a) reformulation of numerical data, b) constraining the new numerical data in short-term memory and c) mental computation of them. Therefore, short-term memory is possibly very important for the process of estimation. Moreover, it is necessary to recall the process of estimation from long-term memory. However, further research is necessary in order to extract safer conclusions.

Data analysis gives evidence that computational estimation ability is a multifactorial phenomenon. The factors that contribute significantly for predicting success at computational estimation concern: a) mathematical background, b) preference to mathematics and c) self-concept.

So, according to the results of the study, there is evidence that a part of computational estimation ability is related to students’ knowledge (mathematical background) and another part of this ability is related to students’ attitude (self-concept, preferences).

However, the multiple regression generated in this study accounted, at best, for 74.9% of the variance in computational estimation performance. Further research is needed: a) to confirm the findings of the present research, b) to investigate other factors that possibly account for the remainder such as training in mental calculation at home or at school, teaching methods and approaches having been used at school, parents’ job, parents’ computational ability or parents’ general mathematical ability.
and c) to investigate the reasons why these factors interact and influence computational estimation ability.

References


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Georgia Chalepaki, Michael Kourkoulos

FACTORS CONTRIBUTING TO COMPUTATIONAL ESTIMATION ABILITY OF PRESERVICE PRIMARY SCHOOL TEACHERS


**BRIEF BIOGRAPHIES**

**Georgia Chalepaki** is a teacher of a primary school. She has graduated from the Department of Primary Education – National and Kapodistrian University of Athens and from the Department of Mathematics – University of Crete. She has received a master and a Ph.D. in Didactic of Mathematics from the Department of Primary Education of the University of Crete.

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