Ἔχεις μοι εἰπεῖν, ὦ Σίκρατες ἀρα διδάκτον ἢ ἄρετή; ἢ οὐ διδάκτον ἀλλ' ἀσκητῶν, ἢ οὔτε ἀσκητῶν οὔτε μαθητῶν, ἀλλὰ φύσει παραγίγεται ταῖς ἀνθρώποις ἢ ἄλλω τινὶ τρόπῳ

A National and International Interdisciplinary Forum for Scholars, Academics, Researchers and Educators from a wide range of fields related to Educational Studies

1st Thematic Issue
Florina, December 2014
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EDITOR'S INTRODUCTORY NOTE

INTRODUCTION TO SPECIAL ISSUE

Behaviour of students, teachers and future teachers in mental calculation and estimation

We are happy to present the first Special Issue of our new journal “MENON: Journal for Educational Research” which was introduced in 2012. Research in Mathematics Education is a significant area of educational research, which is included in the topics of this journal.

“Behaviour of students, teachers and future teachers in mental calculation and estimation” has been chosen as the subject for this special issue on the ground of a number of reasons which are presented below.

Over the past decades, many studies have been conducted in the field of mental calculations and estimation and more precisely in relation to the definition of these concepts, the identification of the strategies used by various age groups, the relationship with other concepts, such as number sense, the procedural and conceptual understanding among others.

Many educational systems have updated the teaching of numbers and operations in mathematics, incorporating mental calculations and estimations in their elementary and middle education curricula.

Nowadays, it is considered timely to conduct research in the implementation of the teaching of mental calculation and estimation with whole and rational numbers as well as the recording of students’ behaviour and the training of pre-service and in-service teachers in these concepts.

During the last decade, researches on mental computation and estimation with rational numbers has been conducted in the Laboratory of “Nature and Life Mathematic” at the University of Western Macedonia, some of which are presented in this issue.

Most of the papers included in this issue, refer to mental calculations and estimations with rational numbers, a topic that is not very common in the literature and covers a wide age range including elementary school students, adults, as well as pre- and in-service teachers. The researches are presented according to the age range of the participants.

- In their study Greet, Bert, Torbeyns, Ghesquière and Verschaffel distinguish between two types of strategies for subtraction: (1) direct subtraction, and (2) subtraction by addition, and provide an overview of the results of 5 related studies using non-verbal methods to investigate the flexible use of these strategies in both single- and multi-digit subtraction. Adults, students and
elementary school students with mathematical learning disabilities have participated in this research.

- Anestakis and Lemonidis in their study, investigate the computational estimation ability of adult learners and implement a teaching intervention about computational estimation in a Junior High School for Adults. They suggest incorporating computational estimation into Second Chance Schools and into adult numeracy teaching practices in general.

- The two papers of Lemonidis, Nolka, Nikolantonakis and Lemonidis, Kaiafa examine the behaviour of 5th and 6th grade students in computational estimation and in mental calculations with rational numbers, respectively. In these studies, the relation between students’ performance in computational estimation and mental calculations with rational number and problem solving ability are also examined.

Four studies on this issue, refer to the prospective elementary teachers’ behaviour in mental calculation and estimation.

- Anestakis and Desli examined 113 prospective primary school teachers’ views of computational estimation and its teaching in primary school. Results revealed that the majority of prospective teachers identified the importance of computational estimation for both daily life and school.

- In their research Kourkoulos and Chalepaki interviewed and examined through a test 69 pre-service teachers aiming to investigate the factors that contribute to their computational estimation ability. They found five factors that contribute to computational estimation, such as the mathematical background and the attitude towards mathematics.

- Lemonidis, Tsakiridou, Panou and Griva used interviews to examine the knowledge and the strategy use of 50 pre-service teachers in multiplication tables and their mental flexibility in two-digit multiplications by using the method choice / no-choice by Lemaire & Siegler, (1995). The results showed that prospective teachers are not flexible in two-digit multiplications and their flexibility to mentally calculate two-digit multiplications is associated with their knowledge in prep and their prep response time.

- Koleza and Koleli have used a test to study the mental computations and estimation strategies of 87 pre-service teachers. The data revealed that the prospective teachers’ number sense concerning rational numbers, basic concepts of the decimal system and elementary numerical properties was very weak.

- Lemonidis, Mouratoglou and Pnevmatikos studied 80 in-service teachers’ performance and strategies in computational estimation and individual
differences concerning their age and work experience, their attitude towards mathematics and their prior performance in mathematics during high school years.

- The last paper of Lemonidis, Kermeli and Palaigeorgiou propose a teaching intervention to sixth grade students in order to promote understanding and enrich their conceptual strategy repertoire to carry out mental calculations with rational numbers. At the same time, three teachers’ attitudes towards teaching mental computation with rational numbers, were examined.

Finally, I would like to thank all the researchers from Belgium and Greece who contributed with their papers in this thematic issue, the colleagues from the laboratory of "Nature and Life Mathematics", the reviewers of the papers and Elias Indos for the organizational and technical support in the journal.

The Editor of the first Special Issue of “MENON: Journal for Educational Research”

Charalampos Lemonidis
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INVESTIGATING PROSPECTIVE ELEMENTARY TEACHERS’ NUMBER SENSE, THROUGH MENTAL COMPUTATION STRATEGIES

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ABSTRACT
The purpose of this study was to explore the number sense of prospective elementary teachers, through a test demanding mental computations and estimation strategies. Eighty seven pre-service teachers from the Department of Primary Education participated in this study. Findings were analysed with quantitative methods. At the end of the research, prospective teachers’ number sense was found to be very low, not only in what concerns rational numbers, but even about the basics of decimal system and elementary numerical properties.

Keywords: Number Sense, Mental Computations, Estimation.

1. INTRODUCTION
Prospective teachers, in Greece, enter to the university having completed secondary education, following (mainly) the General Strand (General Lyceum) or the Technical-Vocational one. Following the General Lyceum, they have to choose among 3 pathways: theoretical, practical and technological one. Most of the prospective teachers usually have chosen the “theoretical pathway”, that means emphasis on language and literature and much less emphasis on Mathematics. Especially during the final (third) year of their secondary studies, given that the general lyceum does not function as an independent and self-contained school but has been transformed into a preparatory level for access to higher education, most prospective elementary teachers have “lost any contact” with mathematics, even of the elementary level. So, even if they have completed secondary education, it is not probable that they can deal with elementary mathematics with understanding. This is rather a worldwide phenomenon, given that many studies have shown that the understanding of elementary mathematics subject content of prospective elementary teachers is “rule-bound and thin” (Ball 1990:449). In a research review, Mewborn (2000) showed that the knowledge of primary school teachers have mainly a procedural knowledge of mathematics and lack the conceptual understanding to provide explanations for rules and algorithms. Other researchers have come to the same conclusion (Alajmi & Reys,
2. THEORETICAL FRAME

2.1. What is Number sense?

Number sense is related to a person’s deep understanding of numbers and operations. Though there are numerous studies about students’ ‘number sense’, the term is not yet defined precisely (Hope, 1989:12). It seems that the first allusion to number sense had been made using the term ‘quantitative intuition’ (Carpenter, Coburn, Reys & Wilson, 1976), while the term was clearly described in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989): “Children with good number sense (1) have well-understood number meanings, (2) have developed multiple relationships among numbers, (3) recognize the relative magnitudes of numbers, (4) know the relative effect of operating on numbers, and (5) develop referents for measures of common objects and situations in their environments” (p. 38). Till then, a number of researchers have described the term, but “no two researchers have defined in precisely the same fashion” (Gersten et al. 2005, p. 296). For Greeno (1991) “number sense is a term that requires theoretical analysis rather than a definition” (p. 170).

Number sense has been described as a mathematical proficiency (Kilpatrick et al., 2001:116) including conceptual understanding of numbers and operations with numbers; procedural fluency to perform operations on these numbers; adaptive reasoning to use different representations and benchmarks to estimate the reasonableness of an answer and strategic competence to apply the knowledge in different contexts and a disposition to make sense of numerical situations. It is a proficiency highly personalized (McIntosh, et al., 1997) with its development being a lifelong process (Reys, Lindquist, Lambdin & Smith, 2007).

There are two main tendencies in the description of the “number sense”:

- a “general description”, and
- a description through the enumeration of the concept’ components.

Examples of the first case are the description of the number sense as a “good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms” (Howden 1989, p. 11), or a “deep understanding of number” (Griffin, 2003:306).

Mcintosh, Reys and Reys (1992) define number sense as “a person’s general understanding of number and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations” (p.3) and as “an ability to use numbers and quantitative methods as a means of communicating, processing, and...
interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity (makes sense)” (p.4).

Concerning the question of “teachability” of the number sense Van de Walle, Bowman and Watkins (1993) see the developing of number sense as a way of teaching rather, than as a body of knowledge and skills to be taught. This is because they see it as having an interconnected, multi-faceted and highly conceptual nature.

The high conceptual nature of number sense and his link with higher order thinking was referred also by Resnick (1989), who lists the following key features of number sense: number sense is non-algorithmic; tends to be complex; often yields multiple solutions, each with costs and benefits, rather than unique solutions; involves nuanced judgments and interpretation and the application of multiple criteria; often involves uncertainty; involves self-regulation of the thinking process and imposing meaning; number sense is effortful (p. 37).

In his literature review (by analyzing forty studies), Berch (2005) found approximately thirty components of number sense, ranging from the ability to compare quantities, to estimate, to the understanding of number meanings and the effect of operations, to the composing and decomposing numbers, to the skill of having a non-algorithmic “feel” for numbers etc.

McIntosh, Reys and Reys (1992) were the first to provide a framework for clarifying and organising the various components of basic number sense. They proposed three key components to number sense: (i) knowledge of and facility with numbers, (ii) knowledge of and facility with operations and (iii) applying knowledge of and facility with numbers and operations to computational setting, as well as their interconnections (p.5). Each one of these components were further analyzed by other researchers (Reys et al. 1999; Tsao 2005; Yang, Reys, and Reys 2009).

For example, knowledge of and facility with numbers has been analyzed in

1. the understanding of the meaning and (relative) size of number (How does 2/5 compare in size to 1/2?) (Behr, Wachsmuth, Post, & Lesh, 1984; Cramer, Post, & delMas, 2002) and
2. the understanding and use of equivalent representations of numbers/Being able to compose and decompose numbers (Show different ways that 2/5 can be represented)
3. Using measurement benchmarks in comparing numbers

knowledge of and facility with operations, has been analyzed in

1. the understanding the meaning and effect of operations (Is 750: 0.98 more or less than 750?) (Greer, 1987; Graber & Tirosh, 1990; Tirosh, 2000)
2. the understanding and use of equivalent expressions

applying knowledge of and facility with numbers and operations, has been analyzed in
1. Flexible computing and counting strategies for mental computation, written computation, and calculators/apply estimation strategies (Sowder, 1992).

2. Judging the reasonableness of computational results

2.2. Number sense and mental computations

Number sense and mental computation are strongly interrelated: In order for students to use “mental computation strategies flexibly requires sound number sense” and when “students have opportunities to work with numbers in flexible ways, provide opportunities for them to improve their number sense. Needing number sense for efficient use of computation strategies and the development of number sense by using such strategies are very closely interrelated” (Hartnett, 2007:345).

For McIntosh and Dole (2000:407), “it appears that mental computation and number sense need to become integral components of curriculum and assessment procedures, at class, school and system levels. Otherwise, the curriculum may be distorted by playing down the importance of number sense and mental computation, and students may be either advantaged or disadvantaged if there is failure to assess important aspects of mathematics”. By many researchers, mental computation is considered as a subset of number sense, given that when students are proficient in mental computation, they also display number sense (e.g., McIntosh, 1996; McIntosh, Reys, & Reys, 1992; Sowder, 1990). Callingham (2005:193) describes research in mental computation as focusing on “identifying and describing students’ strategies for addressing particular kinds of calculations, often within a framework of number sense” (p. 193). Furthermore, specific connections have been proposed among mental computation and aspects of number sense, in particular, number facts knowledge and estimation (Sowder, 1992). Based on this assumption literature has proposed the importance of including mental computation in a mathematics curriculum that promotes number sense (e.g., Klein & Beishuizen, 1994; McIntosh, 1998; Sowder, 1992; Verschaffel & De Corte, 1996). “Curriculum should provide opportunities for students to develop and use techniques for mental arithmetic and estimation as a means of promoting deeper number sense” (Kilpatrick, Swafford and Findell 2001:415).

Nevertheless, other researchers have argued that proficiency in mental computations helps in developing number sense only if students are encouraged to formulate their own mental computation strategies (Blöte, Klein, & Beishuizen, 2000; Sowder, 1990). “Mental computation can facilitate number sense when students are encouraged to be flexible” (Heirdsfield, 2004:443). Even in this case, the high scores in mental computations is not an indice of the number sense. It is the kind of strategies used by the students that guarantees their sense of number: “Students who may score highly on mental computation tests and general mathematics tests may not be developing a "sense" of numbers. And students who do not score highly on written tests of mental computation, number sense and general mathematics may
still have quite good strategies for mental computation and a lot of "sense" about numbers” (McIntosh and Dole, 2004:407).

### 2.3. Number sense of prospective elementary teachers

Although relatively few studies have investigated pre-service teachers’ knowledge of and facility with numbers and operations comparing those conducted in order to investigate students’ number sense, the findings indicate the low performance of teachers. Even if prospective elementary teachers were able to manipulate symbols algorithmically and find mathematical products, they were unable to create intuitive algorithms, arguments, or models that rely on number sense and mathematical reasoning. Kaminski (1997) in a small-scale study (with 6 pre-service teachers) on the use of number sense in the whole number domain, found that teachers preferred using written calculations and rarely utilized estimation, they lacked an understanding of multiple relationships in the number and operations domain and had difficulties with mental computations. Johnson (1998) arrived in the same conclusion: prospective elementary teachers' general number sense are inadequately developed, they resist looking at mathematics in creative, non-algorithmic ways. Tsao (2004) explored the connections between number sense, mental computations and written computations of 155 pre-service elementary school teachers and found that the correct responses on exact computations were higher than those requiring mental computation, estimation or other aspects of number sense. The same researcher (Tsao 2005), in a qualitative research conducted with 12 pre-service elementary school teachers, explored five characteristics of number sense: the ability to decompose/recompose numbers; recognizing the relative and absolute magnitude of numbers; the use of benchmarks; understanding the relative effect of operations on numbers; and flexibility of applying the knowledge of numbers and operations to computational situations (including mental computation and computational estimation). He found that prospective elementary teachers exhibit poor number sense and rely heavily on standard written algorithms. A most focus studies were organized by Alajmi and Reys (2007) and Yang, Reys and Reys (2009). Alajmi and Reys investigated 13 middle school teachers’ capacity to determine the reasonableness of answers. They found that the common view of the teachers of a reasonable answer was an exact answer and that they would use a computational procedure to determine the reasonableness of an answer. The population in the study of Yang, Reys and Reys (2009) was much greater. 280 pre-service elementary teachers were tested about their competency of using benchmarks in recognizing the magnitude of numbers and estimation in knowing the relative effects of an operation on various numbers. Over 60% of the teachers failed to use attributes of number sense - benchmarks and estimation - to produce answers and explain their thinking. Prospective elementary teachers have tended to rely on standard written algorithms, despite the fact that the instructions for these assessments explicitly discouraged such approaches. Şengül, S. (2013) has investigated five different number sense
components. His population was 133 prospective teachers from the Elementary Education Department and the findings were analyzed with qualitative and quantitative methods. Analyzing the results he concluded that pre-service teachers’ number sense was very low, and that pre-service teachers preferred using “rule based methods” instead of “number sense” in each of the components”. In a recent study, Tsao (2012) investigated the number sense of teachers with different backgrounds: mathematics and physics backgrounds, and language backgrounds. He observed that “the teachers with mathematics and physics backgrounds had more complete interpretation towards the related knowledge of number sense. The teachers who did not have mathematics and physics backgrounds but were extremely interested in mathematics had initial understanding towards the related knowledge of number sense. The teachers without mathematics and physics were not interested in mathematics and had less knowledge of number sense” (p.29). A special interest for our study presents the research of Lemonidis and Kaimakami (2013) on prospective elementary teachers’ knowledge in computational estimation. Their main result was that “Greek prospective elementary teachers show a low performance in number sense. […] show a general limited ability in performing mental operations, especially in the case of two-digit number multiplications and in the case of divisions with two-digit divisor “(p.96).

3. METHOD

3.1. Aims of the study

The present study were designed in order to (1) investigate prospective elementary teachers primary teachers number sense as it is defined by (McIntosh, Reys and Reys 1992) and (2) detect the major errors/misconceptions concerning the meaning of numbers and operations on numbers.

3.2. Sample

Participants were 87 prospective teachers in the first year of their studies in University of Patras - Department of Primary Education. The sample consisted of 9 male (10.3%) and 78 female (89.7%) students. The 74 students (85%) were attending in General Lyceum a theoretical pathway, the 6 of them (6.8%) the practical pathway and the rest 7 of them were students having accomplished a circle of studies in another department (8%).

3.3. Instrument development

The design of test items was based on the conception of number sense as an understanding of (McIntosh, Reys and Reys 1992):
   a. the meaning and the relative size of numbers
   b. the meaning and the relative effect of operations on numbers.
c. the different ways of applying knowledge of and facility with numbers and operations

The 28 items were constructed in order to investigate prospective elementary teachers’ number sense, being analyzed in the 4 number sense components as presented in the following table. Prospective teachers had 50’ to answer the test and the only indication given was to «answer with written calculation, only mentally computing». The time restriction was decided to prevent the use of written computations.

<table>
<thead>
<tr>
<th>Number sense components</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1. Understanding of the meaning and (relative) size of number-Sense of orderliness, place value, order numbers Using measurement benchmarks in comparing numbers</td>
<td>Questions 1, 2, 4, 10, 13, 19, 20, 25</td>
</tr>
<tr>
<td>A2. Understanding and use of equivalent representations of numbers (equivalent numerical forms) /Being able to compose and decompose numbers</td>
<td>Questions 7, 8, 9</td>
</tr>
<tr>
<td>B. The meaning and the relative effect of operations on numbers.</td>
<td>Questions 3, 5, 6, 16, 17, 21, 22, 26, 27</td>
</tr>
<tr>
<td>C. The different ways of applying knowledge of and facility with numbers and operations.</td>
<td>Questions 11, 12, 15, 18, 23, 24, 28</td>
</tr>
</tbody>
</table>

4. RESULTS

4.1. Data

The data was analyzed by using SPSS and Microsoft Excel. Every answer was evaluated as right, wrong or no answer. Furthermore, the answers were encoded in “1” for the right ones and in “0” for wrong or no answers. The mean of every answer was computed, by dividing the score in every answer with the whole sample. The results (in frequency and percentage of the right answers) of the ‘number sense’ test are displayed in Table 1 for the total number sense and for the four selected domains.

<table>
<thead>
<tr>
<th>Table 1: Percentages and frequencies of success</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DOMAIN 1: MEANING AND SIZE OF NUMBER</strong></td>
</tr>
<tr>
<td>Q. 1  Q. 2  Q. 4  Q. 10  Q. 13  Q. 19  Q. 20  Q. 25</td>
</tr>
<tr>
<td>Success 21  64  45  38  24  23  23  28</td>
</tr>
<tr>
<td>(24.1%) (73.6%) (51.7%) (43.7%) (27.6%) (26.4%) (26.4%) (32.2%)</td>
</tr>
<tr>
<td><strong>Domain 2: UNDERSTANDING AND USE OF EQUIVALENT REPRESENTATIONS OF NUMBERS</strong></td>
</tr>
<tr>
<td>Q. 7  Q. 8  Q. 9</td>
</tr>
<tr>
<td>Success 39 (44.8%) 31 (35.6%) 59 (67.8%)</td>
</tr>
<tr>
<td><strong>DOMAIN 3: MEANING AND RELATIVE EFFECT OF OPERATIONS ON NUMBERS</strong></td>
</tr>
<tr>
<td>Q. 3  Q. 5  Q. 6  Q. 16  Q. 17  Q. 21  Q. 22  Q. 26  Q. 27</td>
</tr>
<tr>
<td>Success 83  46  55  45  48  42  33  69  53</td>
</tr>
</tbody>
</table>
Eugenia Koleza, Maria Koleli

INVESTIGATING PROSPECTIVE ELEMENTARY TEACHERS’ NUMBER SENSE, THROUGH MENTAL COMPUTATION STRATEGIES

(95.4%) (52.9) (63.2% (51.7% (55.2%) (48.3% (37.9%) (79.3%) (60.9%)

| DOMAIN 4: DIFFERENT WAYS OF APPLYING KNOWLEDGE OF AND FACILITY WITH NUMBERS AND OPERATIONS |
|-----------------------------------------|---------|---------|---------|---------|---------|---------|---------|
| Success | 30 | 33 | 77 | 53 | 36 | 21 | 30 | 57 |
| (34.5%) | (37.9%) | (88.5%) | (60.9%) | (41.4%) | (24.1%) | (34.5%) | (65.5%) |

The mean score of each domain was computed by summing the frequencies of each domain and dividing with the sample (87) and the number of the answers in each domain. After that, the mean of overall score was computed too, by summing all the frequencies and dividing them with the number of all questions (28) and the sample (87). The means’ distributions appear in Figure 1.

Figure 1: Means of Questions

Furthermore, the overall score and the four domains were categorized in 4 values due to the number of the right answers. The selected values are the ‘none’, ‘poor’, ‘average’ and ‘good’.

- None: no right answers
- Poor: right answers in the 1/3 of the questions
- Average: right answers in the 2/3 of questions
- Good: right answers in more than the 2/3 of questions.

The overall score cannot take the value “none” as every member of the sample answered right in more than 4 questions. On the other hand, there were members of the sample that didn’t answer correctly any of the questions in one particular domain,
so the value “none” appears in all the four domains. This type of analysis appears in Figure 2 and Table 2.

**Figure 2:** The relation between the overall score and the four number sense components

![Figure 2: The relation between the overall score and the four number sense components](image)

For example, in the category “Meaning And Relative Effect Of Operations On Numbers”: One student (1.1%) gave no right answers in all the questions of this category and so, her overall score was poor. Seventeen students (19.5%) answered correctly to 1/3 of the questions (‘poor’), and among them fourteen (16.1%) had a ‘poor overall score’ and three (3.4%) had an ‘average overall score’. Thirty eight (43.7%) students had an average perfomance in this domain, as about their overall performance, six (6.9%) had a poor one, twenty nine (33.3%) had an average one and three (3.4%) had a good overall performance. Finally, thirty one (35.6%) students answered good in the questions of this category. Among them, eighteen (20.7%) had an average and thirteen (14.9%) had a good overall performance.

**Table 2:** The relation between the overall score and the four number sense components

<table>
<thead>
<tr>
<th>MEANING AND SIZE OF NUMBER</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NONE</td>
</tr>
<tr>
<td>OVERALL SCORE</td>
<td>3</td>
</tr>
<tr>
<td>POOR</td>
<td>0</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>0</td>
</tr>
<tr>
<td>GOOD</td>
<td>0</td>
</tr>
</tbody>
</table>
The above results allow us to categorize all questions of the test in 4 groups depending on the success in each.

- **Group A** (more than 75%): 3, 14, 26
- **Group B** (50%-75%): 2, 4, 5, 6, 9, 15, 16, 17, 27, 28
- **Group C** (25%-50%): 7, 8, 10, 11, 12, 13, 18, 19, 20, 21, 22, 24, 25
- **Group D** (less than 25%): 1, 23

### Table: Understanding and Use of Equivalent Representations of Numbers

<table>
<thead>
<tr>
<th>OVERALL</th>
<th>NONE</th>
<th>POOR</th>
<th>AVERAGE</th>
<th>GOOD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>POOR</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>6</td>
<td>20</td>
<td>19</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>GOOD</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>16</td>
</tr>
</tbody>
</table>

### Table: Meaning and Relative Effect of Operations on Numbers

<table>
<thead>
<tr>
<th>OVERALL</th>
<th>NONE</th>
<th>POOR</th>
<th>AVERAGE</th>
<th>GOOD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>POOR</td>
<td>1</td>
<td>14</td>
<td>6</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>0</td>
<td>3</td>
<td>29</td>
<td>18</td>
<td>50</td>
</tr>
<tr>
<td>GOOD</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

### Table: Different Ways of Applying Knowledge of the Facility with Numbers and Operations

<table>
<thead>
<tr>
<th>OVERALL</th>
<th>NONE</th>
<th>POOR</th>
<th>AVERAGE</th>
<th>GOOD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>POOR</td>
<td>1</td>
<td>17</td>
<td>3</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>0</td>
<td>20</td>
<td>28</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>GOOD</td>
<td>0</td>
<td>1</td>
<td>10</td>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

The percentages per question are shown in the graph.
Prospective teachers’ percentage of correct responses range from 24.1% to 73.6% in Domain 1 (Meaning and size of number) 35.6% to 67.8% in Domain 2 (Understanding and use of equivalent representations) 37.9% to 95.4% in Domain 3 (Meaning and relative effect of operations) 24.1% to 88.5% in Domain 4 (Applying knowledge and facility with numbers) The overall percentage of correct responses for all ‘number sense’ domains is 50%, actually not that high.

Analyzing teachers’ responses we made two key observations:

1. Half of the items in the questionnaire (15/28) had been answered correctly by less than the 50% of the prospective teachers
2. The mathematical content of these items were:
   - **Positional system:** Q1
   - **Properties of mathematical operations:** Q7, Q8
   - **Multiplication/Division between decimals:** Q18, Q23, Q24
   - **Rational numbers:** Q10, Q11, Q12, Q13, Q19, Q20, Q21, Q22, Q25

None of the questions were answered by more than 50% of the participants. The rest 13 questions were about integers and only two of them were about a division of an integer by a decimal.

Though it appears that prospective teachers performed the best in the “meaning and relative effect of operations” domain (means 61%), in fact, this high mean is due to the questions 3 and 26.

**Question 3** (95.4%): “If you know that 48+37=85, find without using any calculation how much is 49+36?” is a relatively easy ‘algebraic’ question that students deal with during the second grade.

**Question 26** (79.3%),

“Choose the greater of the two:

a) 135+98 or 114+92
b) \( \frac{1}{2} - \frac{3}{4} \) or \( 1 \frac{1}{2} \)
c) 46-19 or 46-17
d) 0,0358 or 0,0016+0,313”

also, deal with simple mental calculations, given that in (a) they could replace 98 and 92 and make mentally the addition, in (b) the comparison of 3/4 and 1 is elementary, in (c) they had to notice that we take a greater result when we subtract a smaller number and in (d) they should notice that in the addition there is the number 0.313 that is greater than 0.0358.

From the other Domains, the questions with the higher success that influenced the means of the domain were:

- For Domain 1 (Meaning and size of numbers),

The **Question 2** (73.6%): “How many notes of 100€ you can have in 5908 €?”. It is a real life situation very familiar to the adult population.
All other questions of the domain had a percentage success below 50%.

- For Domain 2 (Understanding and use of equivalent representations),
  The **Question 9** (67.9%):

  ![Number line diagram](image)

  On this number line, which letter represents better the calculation B + F? Explain why.

  B is approximately 0.2 and F is approximately 2.2. The sum of B+F is lower than 3 so the answer is the letter G. Some of the students answered both G and H because the result should be greater than F.

- For Domain 4 (Different ways of applying knowledge...),

  The numbers in the **Question 14** (88.5%): “The product of which two of the following numbers is closer to 75? 4, 18, 50, 37” lead easily to the correct answer.

  The same thing happens to the Question 28 (65.5%): “Find mentally the products
  a) 18×5×5×2×2  b) 25×62×4  c) 2.5×0×4.

  In (a) the 2X5=10 and in (b) the 4X25=100 has facilitated the answer.

  In the Question 15 (60.9%): “The product of 46×91
  - is greater, equal or lower than 5000?
  - is greater, equal or lower than 3600?”

  In the first part of this task, they had to notice that 46×100= 4600 so the result would be lower than 5000. In the second part of this task, they had to notice that 40×90= 3600 so the result would be greater than 3600.

  All other questions of this domain had a percentage success below 38%.

  We give some examples of the most common mistakes made by the prospective teachers in these 15 low achieving questions. We remind that in all questions we asked for the explanation of their way of thinking.

<table>
<thead>
<tr>
<th>QUESTIONS</th>
<th>COMMON ERRORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) How many tens and hundreds has the number 1457?</td>
<td>5 tens and 4 hundreds</td>
</tr>
<tr>
<td>7) Which of the following calculations has the same result as the product 4×198; a) 4×200 – 2  b) 4×200 – 4  c) 4×200 – 6  d) 4×200 – 8</td>
<td>a) 4×200-2</td>
</tr>
<tr>
<td>8) We know that 93×134 =12462. How much more than 12462 is the product of 93×135?</td>
<td>12465 or 3 more because when we multiple 3 with 5, the product has to end in 5.</td>
</tr>
<tr>
<td>18) Our calculator is broken and it gives 6858 as the result of the product 15.24 × 4.5. Where should we place the decimal point in order to have the correct result?</td>
<td>6.858 because we have 3 decimal digits</td>
</tr>
</tbody>
</table>
### Questions

<table>
<thead>
<tr>
<th>Question</th>
<th>Common Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>23) The product of 0.048 × 0.19 is closer to: a) 0.00009, b) 0.0009, c) 0.009, d) 0.09</td>
<td>a) 0.00009 Because the sum of the decimal digits are five.</td>
</tr>
<tr>
<td>24) The result of 4.2 : 0.33 is closer to: a) 0.012, b) 0.12, c) 1.2, d) 12</td>
<td>a) 0.012 and b) 0.120 because we want 3 decimal digits</td>
</tr>
<tr>
<td>10) Which of the following fractions is closer to 1? 4/5, 6/7, 8/9</td>
<td>4/5 because the whole is divided in bigger parts</td>
</tr>
<tr>
<td>11) Which of the following numbers is closer to the sum 12/13 + 7/8</td>
<td>a) 1 because 12/13 and 7/8 are close to number 1 so the sum would be close to 1.</td>
</tr>
<tr>
<td>12) Which of the following results is closer to 1? A. 5/11 + 3/7, B. 7/15 + 5/13, C. 1/2 + 4/6, D. 5/9 + 8/15</td>
<td>b) 7/15 + 5/12 because these fractions have the biggest denominators.</td>
</tr>
<tr>
<td>13) Use ONLY 2 of the numbers 3, 4, 9, 12, to create a ratio with value closer to 1/2</td>
<td>3/4 because it is really close to 2/4=1/2 and close to 1/2</td>
</tr>
<tr>
<td>19) Compare mentally the fractions below</td>
<td>• 13/28, because we have ‘bigger pieces’</td>
</tr>
<tr>
<td>• 13/28 vs 15/32</td>
<td>• The fractions in both couples are equal, or 4/5 and 15/18 (“bigger pieces”)</td>
</tr>
<tr>
<td>• 8/9 vs 4/5, 16/19 vs 15/18</td>
<td></td>
</tr>
<tr>
<td>20) Which of the fractions below is the greatest? A. 4/5, B. 4/5, C. 499/500, D. 506/501</td>
<td>c) 499/500 because in all fractions the numerator is by 1 less from the denominator, so the fraction with the lower denominator (bigger pieces) is chosen.</td>
</tr>
<tr>
<td>21) Without computing the product 1/2 × 1/4 can you predict if the result is: a) greater than 1/2, b) lower than 1/2, c) greater than 1 d) greater than 3</td>
<td>c) greater than 1 because of the multiplication of fractions that are greater and equal of 1/2</td>
</tr>
<tr>
<td>22) Without actually computing 3/5 : 3/5 can you predict if the result is:</td>
<td>b) lower than 1 because of the division of fractions.</td>
</tr>
<tr>
<td>a) greater than 1 b) lower than 1 c) lower than 1/3 d) greater than 3</td>
<td></td>
</tr>
<tr>
<td>25) The sum of 12/14 + 7/8 is closer to: a) 1, b) 2, c) 19/21, d) 91/100</td>
<td>c) 19/21 because 12/13+7/9=19/22</td>
</tr>
</tbody>
</table>

### 5. Discussion

Have primary prospective teachers a ‘number sense’?

The results of our study lead us to give a rather negative answer to this question. The non-sense of numbers among the prospective teachers becomes particularly obvious by their inability to distinguish between the place-name and the place-value of digits in a number (see Question 1). An explanation of their misconception is probably the one given by Sowder (1997:449), that "place-value instruction is traditionally limited to the placement of digits. Thus, children are taught that the 7 in 7200 is in the thousands place, the 2 is in the hundreds place, a 0 is in the tens place, and a 0 is in the ones place. [...] They do not read the numbers as 7200 ones or 720 tens, or hundreds, and certainly not as 7.2 thousands". The remedial proposed by Sowder in the same paper is -before we begin instruction on decimal numbers-, to provide more instruction on place value with whole numbers, and to practice reading numbers in different ways.

The results have also made apparent prospective teachers’ ignorance of the basic numerical operations. About 55% of teachers consider that 4× 198 = 4× 200 − 2 (Question 7), and 65% of teachers are not able to read 93× 134 =12462 as 134 times 93, so as to conclude that 93×135 is 93 more (Question 8).

Results have also made obvious prospective teachers’ non-sense of decimals and fractions. For example, (Question 24) they are unable to estimate that 0,33 is approximately 1/3, so the result of 4,2: 0,33 is closer to 1,2. For their answer they take into consideration only the sum of the decimal digits as if it was about an operation of addition or substraction.

Furthermore, in their majority, they have no sense of fractions, considering as a condition of equality the “same difference” between the nominator and the denominator. As about the way of adding fractions, the younger students’ common mistake of adding nominators and denominators appear to the prospective teachers also. In fractions’ multiplication they apply the “tacit rule” that “mulitication makes bigger if the multiplier is bigger than the multiplicand”. In the Question 21 they consider the product \(\frac{1}{2} \times \frac{5}{4}\) as bigger than 1/2 because 3/4 is bigger than the 1/2.

**How can teachers help students acquire a “number sense”?** Creating in the classroom an environment that fosters curiosity and exploration at all grade levels, and taking into consideration that number sense cannot be a goal of direct instruction (Greeno 1991:173) it "develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (Howden 1989:11).

But, to be able teachers to help their students acquire a number sense, they must themselves have the “sense” of numbers. The results of this study seems to indicate that this pre-condition is not obvious.

**REFERENCES**


Sowder, J. (1997). Place value as the key to teaching decimal operations. Teaching Children Mathematics, 3(8) 448-453.


**BRIEF BIOGRAPHIES**

**Eugenia Koleza** is Professor of Mathematics Education at the Department of Primary Education, University of Patras, Greece, since 2009. She has graduated from the Department of Mathematics, University of Athens and has acquired her PhD from the IREM of the University Louis Pasteur Strasbourg, France. She is one of the Editors of the journal Critical Science & Education, published by Nissos Pl. Athens and Vice-President of the Hellenic Society of History, Philosophy and Didactics of the Sciences. She is also Editor of the Book Series: Epistemology and Didactics of Mathematics and Natural Sciences - «Liberal Books», Athens. She is the author of four textbooks on Mathematics Education and of a great number of scientific papers. Her main research interest are: Application of Mathematics in Everyday Life, and the Investigation of Psychological and Sociocultural parameters in mathematical teaching and learning.

**Maria Koleli** is an in service teacher. She completed her studies in the Pedagogical Faculty of Primary Education in University of Patras. Her scientific interests are: Mathematics in everyday life, mental calculation and studying the use of technology in teaching/learning of mathematics.