DISCUSSING MATHEMATICAL MODELING CONCERNING PASCAL’S WAGER

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ABSTRACT
We present and analyze teaching work on Pascal's wager realized with Greek students, prospective elementary school teachers, in the context of a probability and statistics course. In this paper we focus on classroom discussion concerning mathematical modeling activities, connecting elements of probability theory and decision theory with elements of philosophical discussions. On the one hand, this link enriched students' scientific culture, and on the other hand, it allowed for deepening the classroom discussion on Pascal's wager.

Keywords: Pascal's wager, prospective elementary school teachers, mathematical modeling, probability theory, decision theory

1. INTRODUCTION

Discussions on philosophical and religious issues have deep and rich historical links with science; this is particularly true about probabilities and statistics (e.g. see Chandler & Harrison 2012, Hacking 1975, Hald 2003, Porter 1986). However, these rich links have been rarely explored in the conventional teaching of these disciplines, and even less (or not at all) at an introductory level.

We argue that: (a) With adequate teaching design and implementation, it is possible to explore such links even with novice students in statistics and probability, (b) Exploring such links can be fruitful, both for the development of students' scientific culture and for the deepening of the discussion with them on the examined philosophical and/or religious issues (see also Kourkoulos & Tzanakis 2015).

To support (a) and (b) above, we present an example of teaching work concerning Pascal's wager that was realized during an introductory seminar on probability and statistics with Greek students, prospective elementary school teachers 1.

1 An initial version of this work was presented in the Science and Religion International Conference (see Kourkoulos & Tzanakis, in press).
In the discussion on Pascal's wager, which has been active for more than three and a half centuries, important elements of scientific culture are involved such as elements of probability theory and decision theory (e.g. see Bras 2009, Hacking 1972, Hájek 2012, Jordan 1994, 2006). However, many of the arguments involved in the discussion on Pascal's wager, although fundamental, can be followed without the need of a sophisticate scientific background; this is related to the fact that the wager was established in the very first period of the historical development of probability theory, and to Pascal’s ingenious way to establish and present his argumentation. This makes these arguments adequate to be accessed by students like ours; however, because of their fundamental character, they have the potential to increase students' interest significantly.

2. BACKGROUND INFORMATION AND FOCUS

Our teaching work was realized during an introductory seminar on probability and statistics (with classroom meetings for 3 hours per week) with 27 4th-year students (25 female and 2 male) in our Department of Education.

Students had a high-school level background in probability and statistics, so the first four weeks were devoted to revising and completing this knowledge (see below). Next, the teacher gave a first presentation on Pascal's wager and asked students to express their thoughts and comments on this issue; the discussion that followed in this way, lasted for four weeks, and constitutes the first part of classroom discussion.

For the second part, the teacher asked students to read an overview of literature on the discussion on Pascal's wager and other relevant reading sources, and to present elements of their personal study in the classroom. The elements presented by the students substantially enriched the classroom discussion; their discussion lasted for three weeks and constitutes the second part of the classroom discussion.

The focus of this paper is to present and analyze some main aspects of the classroom discussion on Pascal’s wager. In particular, the paper aims to present and analyze realized connections between mathematical modeling activities and elements of philosophical reasoning that fruitfully supported both the development of students’ concepts of probability theory and of decision theory, and the evolution of the discussion on Pascal’s wager.

3. TEACHING ON PROBABILITY AND STATISTICS

As already mentioned, our students had a high-school level background in probability and statistics. During their tertiary studies they had not taken any course on probability and/or statistics; however, they had some exposure to readings of statistical results in the context of courses on Pedagogy and Psychology.

2 During these three weeks, four meetings of three hours were realized, instead of three.
Students' knowledge in probability and (descriptive) statistics was revised and completed during the first four weeks. We talked about data organization and their (graphically and numerically tabulated) representation, measures of central tendency (mode, median, mean) and variation (range, interquartile range and standard deviation), the shape of a distribution and skewness. We also talked about the probability multiplication and addition laws, the binomial distribution and examples of its applications (e.g. chance games, wagering situations, simple insurance models) and the Low of Large Numbers and the normal distribution accompanied by adequate examples. Moreover we discussed the concepts of expected value and expected utility, and their differences. Using adequate examples the teacher explained that the criterion of maximum expected utility is more appropriate than the one of maximum expected value for making decisions in wagering situations.

4. FIRST PART OF THE CLASSROOM DISCUSSION

4.1 Introduction and initial debate on Pascal's wager

During the 5th week, the teacher discussed with students on elements of Pascal's life and work (e.g. see Adamson1995, Hacking 1975 ch7-9, Hald 2003 ch5, Mesnard 1951).

Then he gave a first presentation of Pascal's wager. In this context he also mentioned the so-called "many Gods objection" about Pascal's wager.

4.1.1 Many Gods objection

Regarding the "many Gods objection", students agreed that the wager may be meaningless for a person who doubts God's existence but considers that, if He exists, conflicting hypotheses about Him are probable (e.g. he considers that God may be the Holy Trinity, or the 12 Olympian Gods, or Goddess Kali). Students commented that in this case it may be impossible for the person to find a coherent behavior that satisfies all Gods that he considers as probably existing.

However, students considered that if a person (a) doubts God's existence, but (b) still considers that, if He exists, He is an omnipotent, omniscient and...
omnibenevolent God, then such a person may consider the wager meaningful.

During the discussion some students remarked that persons believing (a) and (b) above are more likely to be found in societies with a strong religious tradition, like the Greek society, because in such a society, alternative hypotheses about existing Gods are not supported by the tradition.

4.1.2 God cannot be fooled

A second objection expressed by four students was the following: If someone bets his way of living on the hypothesis of God's existence, as Pascal proposes, and lives a virtuous life but still has doubts about God's existence, then God, as omniscient, will know that he is not a genuine believer and thus this person's efforts will be in vain.

The teacher explained that Pascal doesn't propose the wager to fool God. Pascal believed, he said, that man's heart has the natural tendency to believe in God and the natural ability to perceive that He exists, however because of passions and sins man's heart is blinded and this leaves room for doubts about God's existence. If one accepts the wager and lives a virtuous life, his heart will be purified from passions and sins and thus his heart will perceive God's existence and his doubts will vanish.

Three students commented that if God exists, then the wagering person is not alone in the wager; God is also there and, by appreciating this person's efforts, He may help him by providing whatever feelings or evidence are necessary for that person to genuinely believe in His existence. Four students argued that if God wanted to help in this way for believing in His existence, it would be easy for Him to provide all people with the necessary evidence, and thus atheists or doubting persons would not exist, but this is not the case. One of the previous students answered that God helps to believe in Him those who want to believe, because He respects men's will; a person who wagers his way of living as proposed by Pascal, clearly makes a very strong effort to dissipate his doubts in the direction of believing in God's existence, and thus it is highly likely that he will attract God's help. Five other students as well made comments that endorsed this remark.

4.1.3 Loving and caring unbelievers

A third objection expressed by eight students concerned the idea that unbelievers will lose eternal salvation. Students said that an unbeliever who is a loving and caring person and dedicates his life to help his fellow humans, will not lose eternal salvation, in their opinion, because God been loving and just will not ignore the goodness of his heart and his efforts. Three other students remarked that the church teaches that being a good person is not enough for

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Moreover, three of them commented that this remark also implies that the wager may be less demanding than what the argument of pure heart implies. They thought that perhaps because of God's generosity, He will help the wagering person to believe once He will consider that he makes a strong effort to live a virtuous life and not wait until his heart is fully purified.
eternal salvation; a correct faith is also necessary. However, the first ones persisted in their opinion. Moreover four of them argued that the idea that unbelievers will lose eternal salvation regardless of their goodness is an idea unfair to God, because it presents Him as harsh and intolerant.

### 4.1.4 Selfish motivation

A fourth objection expressed by six students was that if a person that doubts God's existence accepts Pascal's wager only on the basis of Pascal's argument, namely because he doesn't want to lose eternal salvation, then he accepts the wager only because of a self-interested motivation, and it is doubtful that God will reward efforts because of such motivation. A student remarked that in the New Testament eternal hell and eternal salvation are often mentioned as a motive for people to try to be right and avoid sinning; so church does not reject such a motive as a starting motivation for a person to try to ameliorate himself. Three students elaborated on this last point saying that, although such a motivation indeed is not satisfactory, a person that accepts Pascal's wager even on this basis and tries to live a virtuous life, he will perhaps achieve to be gradually liberated from sins and passions; because of this and God's help he may gradually obtain less selfish motives. Thus even with this unsatisfactory initial motivation the wager may have a positive outcome.

**Comment**

In many of the aforementioned students' remarks and considerations, the influence of the Orthodox tradition was obvious, as well as their acquaintance with this tradition.

It is also worth noting that some students' considerations reflected an elaborated thinking in the context of this tradition.

### 4.2 Modeling of Pascal's Wager

After the aforementioned initial debate on Pascal’s wager, the teacher turned the discussion on its modelling. The following table was presented to the students as a summary of the situation faced by the doubting person in the wager.

<table>
<thead>
<tr>
<th></th>
<th>God exists (G.E.)</th>
<th>God doesn't exist (N.G.E.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective probability for G.E. ($p_1$)</td>
<td>Subjective probability for N.G.E. ($p_2$)</td>
<td></td>
</tr>
<tr>
<td><strong>Wager that God exists</strong></td>
<td>Present Life1, Salvation</td>
<td>Present Life2</td>
</tr>
<tr>
<td><strong>Not wager that God exists</strong></td>
<td>Present Life3, Misery</td>
<td>Present Life4</td>
</tr>
</tbody>
</table>

The mathematical modeling demands clarification and a precise statement of initial premises. This demand leads to a re-examination of the initial premises established by philosophical considerations. Often the demanded clarification and precision leads to reconsidering or re-conceptualizing initial
In what follows we present examples of how the demand of mathematical modeling for clarification and precision influenced the consideration of initial premises of Pascal’s wager.

4.2.1 On the partition of hypotheses about God (columns’ partition)

The teacher remarked that Pascal proposed the wager to a hypothetical person doubting God’s existence but considering that if He exists then He is the God as taught by the Christian church, that is, the Holy Trinity. This remark further provoked the discussion on the many Gods objection. Two students said that for a person doubting God’s existence and considering that if He exists, then He is Allah, the wager may also be meaningful; and that this holds also for someone who considers that if He exists He is an omnipotent, omniscient and omnibenevolent God, without specifying His name and religion. Five other students made similar comments agreeing with their colleagues. Three students remarked that although the wager may be meaningful for such a person, his efforts may be in vain because he wagers in a wrong faith. Four students argued that, following the church, believing in the Holy Trinity is a condition for salvation only for those who have been properly taught the Gospel; thus, for example, for a doubting person that lives in an Islamic society and has not been taught the Gospel this objection doesn't hold. Three students argued that in all these cases, if the wagering person achieves to live a virtuous life and obtain pure heart, then if the pure heart argument holds, he will perceive that He exists, and with His help he will end up with whatever faith He considers adequate for his salvation; so in all these cases the wager may have a positive outcome.

4.2.2 On the partition of possible courses of action (rows’ partition)

The teacher recalled that Pascal argues that wagering about God’s existence is not optional for a doubting person; so he doesn't distinguish between those who don't wager that God exists and those who wager that God doesn't exist.

Six students argued that it would be better if the line "Not wager that God exists" was split into two lines: "Not wager that God exists and live a virtuous life" and "Not wager that God exists and not live a virtuous life". Four students considered that it would be better to split the other line into two too; "Wager that God exists and achieve to live a virtuous life" and "Wager that God exists but do not achieve to live a virtuous life".

4.2.3 Reconsideration of the wager about God’s existence

These remarks led three students to comment that the wager should be adapted to the beliefs of the different categories of persons that doubt God's existence. Two students went further to propose that the wager should be personalized in order to be adapted to the beliefs of each person who doubts God’s existence. Many other students (11) made comments endorsing these
considerations. Thus, the idea emerged in the classroom that the wager about God’s existence should be regarded as personal and be adapted to each doubting person’s considerations and beliefs.

This was an important idea that emerged during the first part of the mathematical modeling work on the wager; that is the clarification of the initial premises of the modeling.

This new consideration of the wager about God’s existence was later developed further. In the context of this reconsideration of the wager, Pascal’s wagering proposal was considered as a special case that initiated the discussion and as a point of reference for establishing alternative versions of the wager adapted to each doubting person’s beliefs.

### 4.2.4 Other initial premises for modeling Pascal’s wager

The teacher told the students that it would be interesting to examine such variants of Pascal’s Wager, but after the examination of the initial version, which was done later. Subsequently, the teacher commented that in the wager’s text Pascal attributes explicitly positive infinite utility to Salvation (“an infinity of an infinitely happy life”, see Pascal 1910: 85), while he is not explicit about the negative utility of Misery. However, he said, Pascal was a devoted Catholic and his hypothetical doubting person considers that if God exists, He is as taught by the Church. Therefore, he said, we may examine first the most severe version of the wager where Misery has infinite negative utility (eternal damnation, eternal hell); this version accentuates the dilemma faced by the doubting person. The teacher also remarked that, according to Pascal, all Present Lives (1, 2, 3 and 4) have finite utility value, because they all have finite time and finite pleasures and displeasures.

He also mentioned that \( p_1, p_2 \) are the probabilities that the doubting person attributes to the hypotheses that God exists or not; thus they pertain to subjective probabilities\(^8\). However, he added, at this early time neither the relevant concepts of probability theory, nor the corresponding terminology had been formulated; thus Pascal explains his idea through examples of relevant betting situations. Pascal’s examples were also discussed with the students.

### 4.2.5 Argument from dominance

Subsequently, the teacher remarked that Pascal argues that for the present life, wagering in favor of God’s existence and living a virtuous life is better and in fact more pleasant than wagering that God doesn’t exist and not live a virtuous life. Thus, according to this, the utility value of Present Life2 is greater than the utility value of Present Life4 and the same holds for Present Life1, compared to Present Life3 (\( U(PL_2) > U(PL_4) \) and \( U(PL_1) > U(PL_3) \)). If a doubting person agrees with this, then for him it is advantageous to wager that God exists in both eventualities (God exists or not).

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\(^8\) He also recalled that \( p_1, p_2 \) are not 0 or 1 and \( p_1 + p_2 = 1 \).
The teacher also remarked that this argument of Pascal is often called an argument from dominance; in the sense that one choice (here, wagering in favor of God’s existence) is advantageous (dominates) in all possible eventualities (here, God exists, or not); e.g. see Hacking 1972.

Students agreed that if a doubting person agrees with this consideration, in addition to all previous hypotheses about his beliefs, then it is reasonable that he will consider advantageous for him to wager that God exists. However, they remarked that there are too many hypotheses on the beliefs and considerations of the hypothetical doubting person, and this makes important the question of whether there are such real persons. Some of them also said that many doubting persons may consider such a virtuous life as the one proposed by Pascal, harsh and unpleasant; so, they concluded, perhaps this last hypothesis holds only for very few.

4.2.6 Argument from dominating expectation

Then the teacher remarked that for those who do not agree with the last hypothesis (that \(U(PL_2)>U(PL_3)\) and \(U(PL_1)>U(PL_3)\)) Pascal proposes another argument:

The expected utility of wagering that God exists is

\[E_1 = p_1 \cdot (+\infty + U(PL_1)) + p_2 \cdot U(PL_2) = +\infty \] (since \(0<p_1<1\), \(0<p_2<1\))

The expected utility of wagering that God doesn’t exist is

\[E_2 = p_1 \cdot (-\infty + U(PL_3)) + p_2 \cdot U(PL_3) = -\infty,\]

so \(E_1\) is greater than \(E_2\), even if \(p_1\) is very small.

The rational choice for wagering is the choice with the greater expected utility\(^9\), which in this case is that God exists.

During the formation and examination of these mathematical equations students:

(i) encountered and worked with infinite expected utilities, which is a concept important both in probability theory and in decision theory,

(ii) encountered, discussed and applied the principle of maximum expected utility, which is an important criterion for decision making in decision theory,

(iii) had the opportunity to understand that the mathematical modeling of Pascal’s wager suggests that a doubting person has to wager in favor of God’s existence, even if the probability that he attributes to the eventuality that God exists is very small.

4.2.7 Contrasting results of mathematical modeling with pragmatic considerations

Subsequently, the teacher asked students to comment on the results of the mathematical modeling of Pascal’s argument, which is based on the danger of losing eternal salvation and suffering eternal hell.

\(^9\)This criterion for wagering and more generally for making decisions is often called the principle of maximum expected utility and it is an important element examined in decision theory. (As Hacking (1972) remarks, Pascal is the first who annunciated this and other important elements of decision theory.) The argument based on this criterion is often called the argument from dominating expectation.
The students initially thought that this argument should be logically convincing for Pascal's targeted audience (persons who doubt God's existence but believe that if He exists then the teaching of the Church about Him is correct). Subsequently, they remarked that all those who consider Church's teaching to be true agree with Pascal's consideration that there is a danger of losing eternal salvation and suffering eternal hell. However, they remarked, a considerable number of these persons, despite of this belief, make very little effort to live a virtuous life. So since the argument based on this danger does not convince many persons who believe that the danger is true, then the argument may also not convince doubting persons to whom Pascal is addressed.

Students continued discussing about why the argument, despite the fact that it seems rationally powerful, does not convince many persons who believe that the danger to lose eternal salvation is a true danger. Students proposed different explanations; one of these that attracted the attention and interest of many students is the following: People find it very unpleasant and painful to think of the eventuality that they will lose eternal salvation and will suffer eternal hell; thus they avoid thinking about it and most of the time, or even all the time, they live their lives without thinking about this eventuality.

Three students remarked that this is not specific to the danger of suffering eternal hell and losing eternal salvation; it is part of a more general behavior of people that concerns avoiding thoughts about extremely negative (either certain or probable) future events. For example, they mentioned that most people avoid and think rarely about their own death or the death of their (living) parents, which are certain future events, because such thoughts are very painful and hard. Six students gave other examples endorsing this consideration, such as avoiding thinking about future illnesses, accidents, professional catastrophes etc. However, four students commented that although existent indeed, such a behavior may become irrational when someone avoids thinking about eventualities such as professional catastrophes or some kind of illness or even suffering eternal hell, because these are cases for which, if he thinks, he can take action to minimize the risk of negative outcomes. Nevertheless, remarked one student, if someone thinks about suffering eternal hell not superficially, but intensively, and uses his imagination in order to catch even a small part of what he may suffer there, then such thoughts quickly become totally unbearable. Five other students commented that if someone frequently or - even worse - continuously thinks about things such as losing eternal salvation and suffering eternal hell, his future death, and so on, he may easily make his present life really miserable by his own thoughts alone. Two of them also commented that the aforementioned avoidance behaviors are in fact important self-protection behaviors. Four other students made comments arguing in favor of this

10 Other explanatory elements proposed by students (such as that there are Christians who don't believe in eternal hell, or that there are people, like drug addicts, who have no more strength to be liberated from their passions) engendered limited discussion in the classroom at that time.
consideration.\textsuperscript{11}

Students agreed that these avoidance and self-protection behaviors may very well be a strong explanatory factor of why Pascal's argument based on the danger of losing eternal salvation and suffering eternal hell is less convincing than he thought; and that this explanatory factor also concerns the relevant version of the argument for those who believe that the teaching of the Church is true.\textsuperscript{12} They also agreed that for persons who avoid considering the danger of losing eternal salvation, mathematical modeling which attributes infinite utility value to salvation and damnation, like the one already mentioned, is inadequate for representing their questions and dilemma about God and His existence.

4.3 Comment

In the first part of classroom discussion, the students acquired some familiarity with Pascal's wager and its mathematical modeling and discussed basic objections about the wager at an initial level. During the modeling of Pascal's wager they had the opportunity to encounter and work with infinite expected utilities. Moreover they encountered, discussed and applied the principle of maximum expected utility.

Furthermore they realized some significant advances concerning the conceptualization of Pascal's wager.

They considered that the wager about God’s existence should be regarded as personal and thus be adapted to each doubting person’s considerations and beliefs. In this context Pascal’s wagering proposal was considered as a special case that initiated discussion, and as a point of reference for shaping alternative versions of the wager.

Students examining Pascal's argument which is based on the danger of losing eternal salvation and suffering eternal hell, considered, on pragmatic grounds, that it has not the convincing power that Pascal thought it had. This, in turn, led them to question the adequacy of Pascal's utility function about eternal salvation and eternal hell.

5. SECOND PART OF CLASSROOM DISCUSSION

Preparing for the second part of classroom discussion, in the 7\textsuperscript{th} week of the course, the teacher proposed that students read an overview on the debate on Pascal's wager (Hájek 2012) and some other relevant writings (in particular Hacking 1975, Jordan 1994, Lycan & Schlesinger 1989). He encouraged them to

\textsuperscript{11} Moreover, three students remarked that considerations of the kind “I live my life now, I repent later” may facilitate the avoidance wished because of self-protection mechanisms. Four students argued that frequently suffering the thought of the threat of eternal hell may produce in certain people worst attitudes than avoidance; such as rejecting altogether Church and its teaching.

\textsuperscript{12} It is interesting to note that these students' considerations are in line with well known pastoral considerations and concerns about the convincing power and the role of arguments based on the danger to loose eternal salvation and suffer eternal hell (e.g. see Bishop Kallistos Ware 1998, p.6).
feel free, after this initial reading, to continue focusing on authors or lines of thought that they would find interesting and attractive in relation to their own ideas and thoughts. The students actively worked on this task as they found the subject attractive. So, from the 9th to the 11th week of the course they orally presented in the classroom elements of their study and their own comments that substantially enriched the discussion there. Below we describe some characteristic aspects of this second part of classroom discussion:

Students encountered in their readings and presented in the classroom, a spectrum of hypotheses about God significantly larger than the one that they considered in the first part of classroom discussion. For some of these hypotheses they thought that they are only intellectual constructs elaborated for the sake of argument, or that it is improbable (or very rare) to be hypotheses having some significant weight in the considerations of real doubting persons; for example, because they totally lacked the support of tradition. However they found others interesting, in particular those hypotheses that suggest that there is no eternal hell such as the hypothesis that all will be finally saved, or the hypothesis that after death the righteous are saved and the wicked pass to nothingness, not to eternal hell. For this last hypothesis they even formulated a corresponding version of the wager and its mathematical modelling. For this version students considered the utility value of salvation to be $+\infty$ and the utility value of hell to be 0.

Students also discussed Penelhum’s (1971: 211-219) objection that the consideration of Pascal’s wager that honest unbelievers will lose eternal salvation is an immoral consideration. This enriched and deepened the previous relevant discussion in the classroom (see section 4.1). Moreover, in relation to this discussion, the teacher along with the students examined the mathematical modeling of a version of the wager with the additional assumption that virtuous doubting persons who don’t wager in favor of God’s existence do not lose eternal salvation.

5.1 Duff’s objection

Moreover, two students presented Anthony Duff’s (1986) objection on Pascal’s wager that a doubting person who does not wager in favor of God’s existence still has some chance to convert before the end of his days. During the discussion on this objection, four students argued that a person who in the present wagers in favor of God’s existence and tries hard to live a virtuous life, still is not certain about eternal salvation because he may fall even at the end of his life, and conversely, it is not certain for a person who wagers against God’s

13 The two weeks of Easter holiday were between the 8th and the 9th week of the course.
14 For example, the hypothesis of Martin (1983) that God rewards the unbelievers and punishes the believers, or the hypothesis of infinitely many possible Gods. It is worth noting that students’ arguments for restricting the spectrum of hypotheses to be considered find support in some of Lycan and Schlesinger considerations (see Lycan & Schlesinger 1989, Schlesinger 1994)
15 This version concerns a person that doubts God’s existence and believes that if He exists, then this hypothesis is true.
existence and lives a non-virtuous life that he will suffer eternal hell because he may repent even at the end of his life\textsuperscript{16}. Seven other students made comments that endorsed these considerations. Moreover three of them suggested that the modeling of the wager should allow for some probability of suffering eternal hell for persons who at present wager in favor of God’s existence, and some probability of obtaining salvation for those who at present wager against God’s existence.

A relevant version of the wager was modeled with teacher’s help\textsuperscript{17}. In this version, both the expected utilities of wagering in favor of God’s existence and against God’s existence were undetermined; so the application of the criterion of \textit{maximum expected utility} was inconclusive. These results initially puzzled students. After further examination six of them considered that since the criterion of \textit{maximum expected utility} was inconclusive then the doubting person should consider that the odds of eternal salvation are greater in the case of wagering in favor of God’s existence and the converse holds for the odds of suffering eternal hell; and that this consideration points in the direction of wagering in favor of God’s existence\textsuperscript{18}. It is worth noting that with these comments students proposed to use a decision-making criterion of \textit{maximum probability} similar to that proposed by Schlesinger (1994)\textsuperscript{19}.

Four other students, based on grounds of intuitive rationality, thought that the difference of the Expected utility of wagering in favor of God’s existence minus this one of wagering against God’s existence is \( + \infty \); and that this also points to the direction of wagering in favor of God’s existence. However, three other students objected that concluding that one undetermined value is better or greater than another undetermined value is meaningless, and thus the conclusion should be that this modeling leads to no definite conclusion. The discussion on this issue permitted students to understand that although there are criteria according to which this modeling leads to conclusion, they are controversial.

After this discussion, the teacher discussed with students relevant paradoxes involving utilities and expected utilities of infinite value\textsuperscript{20}.

\textsuperscript{16} These students’ remarks echoed the well known Church’s teaching that no-living person can be sure about his salvation after death.

\textsuperscript{17} In this version, the utility values of eternal salvation and of suffering eternal hell were considered, once again, to be \( + \infty \) and \( - \infty \) respectively. The conditional probabilities of eternal salvation and of suffering eternal hell, if God exists and the doubting person’s wagers in favor of God’s existence, were named \( p_s, p_h \); both \( p_s, p_h \) were considered to be different than \( 0 \) and \( p_s + p_h \) was considered to be equal to \( 1 \). The respective conditional probabilities if God exists and the doubting person’s wagers against God’s existence were named \( p'_s, p'_h \); both \( p'_s, p'_h \) were considered to be different than \( 0 \) and \( p'_s + p'_h \) was considered to be equal to \( 1 \). It was also considered that \( p_s > p'_s \) and consequently \( p_h < p'_h \).

\textsuperscript{18} In their argumentation, they considered that utilities and expected utilities of earthly lives could be disregarded in this modelling because of being too small, compared to the infinite utilities and expected utilities of salvation and hell.

\textsuperscript{19} Which, however, is not uncontroversial (e.g. see Bartha 2007, Sorensen 1994).

\textsuperscript{20} Some of them concerned the wager, while others did not; the teacher also suggested further relevant reading (e.g see Bartha 2007, Jordan 2006 ch4, Sorensen 1994).
5.2 Finiteness of human perception and understanding

Concerning the utility value of hell and salvation, three students presented a relevant consideration that they had read about; that, although salvation and hell may be infinite, humans may not be able to appreciate this infiniteness adequately because their perception and understanding are finite in several respects (Hájek 2012). Many (12) students endorsed this consideration and argued that living humans are able to perceive eternal salvation and suffering eternal hell only at an abstract level and not at the level of feelings and sensations. Four of them stretched that what Pascal proposes for salvation (an infinity of infinitely happy life) can not be perceived because man has neither the experience of happiness of infinite intensity nor the ability for this feeling; and that the same holds for feelings of suffering of infinite intensity.

However, seven students remarked that a mathematical modeling of the wager which attributes finite utility values to salvation and damnation is not satisfactory with regard to men's ability to perceive infinite utilities for salvation and damnation, even though at an abstract level only. Three of them also argued that for persons who believe that if God exists then the teaching of the Church is true such a modeling does not represent their beliefs and considerations. Five students commented that since men cannot perceive such infinite utilities at the level of feelings and sensations but can do so at an abstract level, then, both modelling with finite such values and modelling with infinite ones will be unsatisfactory with respect to one or to the other.

Four students argued that, although the aforementioned considerations about the finiteness of human perception and understanding are reasonable, previous modeling involving infinite utility for eternal salvation and hell should not be considered as invalid because of these considerations, since humans can still conceive such utilities, even though at an abstract level only. They thought that such modeling should be available to people that consider it adequate for themselves; for instance, persons who consider that argumentation of this kind is very important to them.

Following these considerations, students, with the teacher’s help, formulated a relevant version of the wager and its mathematical modelling. In this version they considered the utility values of salvation and of suffering hell to be finite. Students observed that in this version of the wager the application of the criterion of maximum expected utility is possible to suggest not to wager in favor of the hypothesis of God's existence, and that this depends on the considered utility and probability values. They thought this to be another important difference from previously examined versions of the wager. Eight students considered that in this version of the wager the utility values are closer to the reality of limitations of human understanding. Six of them argued that because of this the possible outcomes of the criterion include the alternative result (not wager in favor of God's existence) which is also a real behavior.

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21 Pascal, remarked two of them, should be one such person.
observed among doubting persons.

6. FINAL COMMENTS

The classroom discussion and students’ related individual work realized during this course allowed them to gain some significant insights into Pascal’s thought about the wager concerning God’s existence, as well as on the relevant debate among philosophers and decision theorists. Moreover they realized some significant conceptual advances concerning this subject.

- They reconsidered Pascal's wager in a dynamic way. More precisely they considered that wagering about God's existence should be considered as personal and be adapted to each doubting person’s considerations and beliefs. In this context, the initial version of the wager was regarded as a special case that initiated the subject and as a reference point for shaping alternative versions of the wager.

- Students considered, on pragmatic grounds, that Pascal’s argument based on the danger of loss of eternal salvation has less convincing power than what Pascal had thought. This also led them to question the adequacy of infinite utility values attributed to salvation and damnation in the context of the corresponding mathematical modeling.

- In connection with the aforementioned, students worked on the modeling of different versions of the wager. This permitted them to work with the concepts of infinite utility and infinite expected utility (concepts which they had very little familiarity with until then) as well as face some interesting problems of decision theory in situations that such utilities are involved.

6.1 Students’ familiarity with Orthodox tradition and the discussion on Pascal’s wager

All along the classroom discussion, in students’ comments and considerations, their familiarity with Orthodox tradition and the important influences they have received from this tradition, were frequently observed.

Students’ relation to the Orthodox tradition both restricted and deepened important aspects of the discussion on Pascal’s wager. This is particularly true with reference to (i) the many Gods objection on Pascal’s wager, and (ii) students’ comments on doubting persons’ considerations concerning God’s existence.

Their relation to the Orthodox tradition was a factor that works in the direction of restricting the spectrum of hypotheses about God that they considered interesting to examine as hypotheses of persons doubting God’s existence. A number of such hypotheses, which were put forward by...

22 However, given the extent and the importance of this debate, the work done in this course has to be considered only as a first-initiation work on Pascal’s wager.
philosophers and decision theorists, were considered by the students as uninteresting to be examined, because they lacked the backup of tradition and were thought of as improbable (or very rare) to be hypotheses that have some significant weight in the considerations of real doubting persons. On the other hand, their relation to this tradition was a factor that enriched and deepened their thoughts on the hypotheses that they examined. Moreover, students’ relation to the Orthodox tradition enriched the insightfulness of their thinking concerning doubting persons’ considerations about God’s existence.

6.2 Mathematical modeling in the discussion on Pascal’s wager

In the class work on Pascal’s wager, elements of probability and decision theory were systematically involved. Besides (subjective) probabilities, utilities and expected utilities, often of infinite value, were involved as well as criteria of decision-making.

These elements were structured in modelling activities of versions of Pascal’s wager and led to interesting problems of decision theory. The mathematical elaboration on infinite values already presented some difficulty for students; but more importantly, often the results of mathematical elaboration were questionable or even in contrast with respect to intuitive rationality. Such tensions enhanced or led to questioning the initial premises of the modeling, for example, questioning the adequacy of the attribution of infinite values to involved utilities and expected utilities. However, replacing these infinite values with finite ones presented other fundamental inadequacies. Thus, in these modeling activities students encountered and worked with the concepts of utilities and expected utilities of infinite value and faced some related questions which are deeply routed in probability theory and decision theory, along with a network of relevant problems.

In these modelling activities, students observed that correct mathematical elaboration does not always lead to safe and/or uncontestable results; as it is, for example, the case in Euclidean Geometry, where the initial premises (axioms) are not questioned. On the other hand, the clarity of mathematical elaborations that led to question initial premises of the modelling permitted to identify flaws of these premises that it was very difficult or impossible, to identify as long as these premises were discussed at the literal level.

Thus, these modelling activities offer students the occasion to appreciate that mathematics may have an important role in the discussion of philosophical issues, to understand some basic aspects of modelling work and even to question stereotypes and enrich their concept image for mathematics.

REFERENCES

23 Although they had heard about the existence of non-Euclidean Geometries, students had never worked with Geometry which was incompatible with the Euclidean one. Moreover, students had very little, if any, experience of mathematical modelling work that may lead to unsafe or contestable results for reasons other than the well known “you haven’t done your work correctly”.


Bishop Kallistos Ware (1998). Dare we hope for the salvation of all? Origen, St Gregory of Nyssa and St Isaac the Syrian. Theology Digest, 45(4), 303-317.


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24 This translation was reissued by Dover Publications in 2003, under the title Pensées. The reissue includes an introduction by T. S. Eliot, written in 1958.
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