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EDITOR'S INTRODUCTORY NOTE

INTRODUCTION TO SPECIAL ISSUE

Behaviour of students, teachers and future teachers in mental calculation and estimation

We are happy to present the first Special Issue of our new journal “MENON: Journal for Educational Research” which was introduced in 2012. Research in Mathematics Education is a significant area of educational research, which is included in the topics of this journal.

“Behaviour of students, teachers and future teachers in mental calculation and estimation” has been chosen as the subject for this special issue on the ground of a number of reasons which are presented below.

Over the past decades, many studies have been conducted in the field of mental calculations and estimation and more precisely in relation to the definition of these concepts, the identification of the strategies used by various age groups, the relationship with other concepts, such as number sense, the procedural and conceptual understanding among others.

Many educational systems have updated the teaching of numbers and operations in mathematics, incorporating mental calculations and estimations in their elementary and middle education curricula.

Nowadays, it is considered timely to conduct research in the implementation of the teaching of mental calculation and estimation with whole and rational numbers as well as the recording of students' behaviour and the training of pre-service and in-service teachers in these concepts.

During the last decade, researches on mental computation and estimation with rational numbers has been conducted in the Laboratory of “Nature and Life Mathematic” at the University of Western Macedonia, some of which are presented in this issue.

Most of the papers included in this issue, refer to mental calculations and estimations with rational numbers, a topic that is not very common in the literature and covers a wide age range including elementary school students, adults, as well as pre- and in-service teachers. The researches are presented according to the age range of the participants.

- In their study Greet, Bert, Torbeys, Ghesquière and Verschaffel distinguish between two types of strategies for subtraction: (1) direct subtraction, and (2) subtraction by addition, and provide an overview of the results of 5 related studies using non-verbal methods to investigate the flexible use of these strategies in both single- and multi-digit subtraction. Adults, students and
elementary school students with mathematical learning disabilities have participated in this research.

- Anestakis and Lemonidis in their study, investigate the computational estimation ability of adult learners and implement a teaching intervention about computational estimation in a Junior High School for Adults. They suggest incorporating computational estimation into Second Chance Schools and into adult numeracy teaching practices in general.

- The two papers of Lemonidis, Nolka, Nikolantonakis and Lemonidis, Kaiafa examine the behaviour of 5th and 6th grade students in computational estimation and in mental calculations with rational numbers, respectively. In these studies, the relation between students' performance in computational estimation and mental calculations with rational number and problem solving ability are also examined.

Four studies on this issue, refer to the prospective elementary teachers' behaviour in mental calculation and estimation.

- Anestakis and Desli examined 113 prospective primary school teachers' views of computational estimation and its teaching in primary school. Results revealed that the majority of prospective teachers identified the importance of computational estimation for both daily life and school.

- In their research Kourkoulos and Chalepaki interviewed and examined through a test 69 pre-service teachers aiming to investigate the factors that contribute to their computational estimation ability. They found five factors that contribute to computational estimation, such as the mathematical background and the attitude towards mathematics.

- Lemonidis, Tsakiridou, Panou and Griva used interviews to examine the knowledge and the strategy use of 50 pre-service teachers in multiplication tables and their mental flexibility in two-digit multiplications by using the method choice / no-choice by Lemaire & Siegler, (1995). The results showed that prospective teachers are not flexible in two-digit multiplications and their flexibility to mentally calculate two-digit multiplications is associated with their knowledge in prep and their prep response time.

- Koleza and Koleli have used a test to study the mental computations and estimation strategies of 87 pre-service teachers. The data revealed that the prospective teachers' number sense concerning rational numbers, basic concepts of the decimal system and elementary numerical properties was very weak.

- Lemonidis, Mouratoglou and Pnevmatikos studied 80 in-service teachers’ performance and strategies in computational estimation and individual
differences concerning their age and work experience, their attitude towards mathematics and their prior performance in mathematics during high school years.

- The last paper of Lemonidis, Kermeli and Palaigeorgiou propose a teaching intervention to sixth grade students in order to promote understanding and enrich their conceptual strategy repertoire to carry out mental calculations with rational numbers. At the same time, three teachers’ attitudes towards teaching mental computation with rational numbers, were examined.

Finally, I would like to thank all the researchers from Belgium and Greece who contributed with their papers in this thematic issue, the colleagues from the laboratory of "Nature and Life Mathematics", the reviewers of the papers and Elias Indos for the organizational and technical support in the journal.

The Editor of the first Special Issue of “MENON: Journal for Educational Research”

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ABSTRACT

Problem solving and number sense are two of the core subjects on which strong emphasis is given in contemporary mathematics curricula of compulsory education. In this study, we examined fifth and sixth grade Greek students’ number sense concerning the mental calculation with rational numbers and specifically fractions and percents. We attempt to analyze the behaviors of fifth and sixth graders in mental calculations with fractions and percent examining the performance of students, categorizing the mental strategies used by them. Despite of the educational importance of these two mathematical areas – problem solving and number sense in mental calculations with rational number – there are not studies which examine directly the relation of students' performance in these two areas. This study has shown that the majority of students use rule-based strategies in operations with fractions and percents. Another result is that the students’ strategy choice (number sense or rule based) relates to their performance in problem solving.

Keywords: number sense, rational numbers, problem solving.

1. INTRODUCTION

The rational number concept is one of the most important, but also more complex mathematical concepts that children will experience during their primary school years. Empirical studies conducted in different countries have highlighted the difficulties faced by students. Students seem to respond well to the use of algorithms, but they lag behind in understanding the concept of fractional numbers and solving verbal and realistic problems involving fractions (Aksu, 1997; Dufour-Janvier, et al., 1987; Kerslake, 1986; Lesh, et al., 1987; Mack, 1995; Nunes, and Bryant., 2009;
Stafylidou & Vosniadou, 2004; Thompson, & Saldanha, 2003; Vamvakoussi & Vosniadou, S., 2010).

An integral part of number sense is the mental calculation in general and mental calculation with rational numbers in particular (Reys, R. 1984, Reys, B. 1985). Mental calculation with rational numbers contributes to a deeper understanding of rational numbers and their operations. Mental calculations based on conceptual understanding of numbers and operations and on a holistic approach to numbers (McIntosh, 2006). In this research, we examined a sample that consisted of Greek students in fifth and sixth grade primary school. Although Greek curricula include mental calculations with whole numbers, however, they only make a general reference and a piecemeal presentation of mental calculations with rational numbers, with emphasis on estimation procedures. Greek curricula and textbooks do not provide a specialized teaching proposal regarding mental calculations with rational numbers. In addition, teachers do not know the strategies as they are not trained in this particular subject. Therefore, it is important to investigate the Greek students’ number sense, regarding mental calculations with fractions and percents and study students’ strategies and their errors in operations with fractions and percents.

Yang (2005) in his research with sixth grade Taiwanese students, regarding the number sense for whole and decimal numbers found that these students were inclined to use ‘rule-based methods’ or ‘could not explain’ when responding to interview questions. They didn’t know how to explain their methods. In our research, in each task, we ask students to explain in writing how they have calculated. We do this in order to understand the strategies of students, but also to examine their ability to understand the operations which they have used. Problem solving has occupied a central position in school mathematics curricula and many studies have been conducted on this subject (Schoenfeld, 1992, 2007). Many researchers underline the correlation between mental calculation and problem solving (Reys, 1984, Thompson, 1999). However, there has not been specific research investigating the relationship between students’ ability in problem solving and their performance in mental calculations with fractions and percentages.

There is a research done by Louange, & Bana, (2010) with Year 7 students, which aim was to determine the relationship between students’ number sense and their problem-solving ability by means of paper-and pencil tests, classroom observations, and interviews of students and teachers. The results revealed a strong correlation between these two aspects of school mathematics. Wai and Kheong (1998) examined the correlation between problem-solving ability, mental calculation ability and estimation ability of primary 5th grade pupils in Singapore. Results from quantitative analyses showed that the scores of problem solving, mental calculation and estimation were significantly correlated.

In this paper, we try to contrast the ability in mental calculations with fractions and presents, with the problem-solving ability.
2. THE STUDY

The purpose of the present study is twofold: a) to examine the students’ number sense concerning mental calculations with fractions and percents, namely, to examine the performance and errors of students in operations, to record the strategies which the students use when performing mental calculations with fractions and percents, and b) to examine the relation between students’ mental calculations ability with fractions and percents and their problem solving ability.

2.1. Number sense in fractions and percents

What is number sense? It has been more than half a century since the concept of number sense was delimited (Dantzing, 1954) referring to the abilities of use and understanding of numbers, arithmetic relations and operations. Nevertheless, the number sense is difficult to define, so various researchers have identified several combinations that include a conceptual understanding of the number and specific numerical skills as components of number sense (Berch, 2005). Quoting McIntosh, Reys, & Reys (1992), “Number sense refers to a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations.” (p. 3).

However, several researchers have reasoned that many students in the middle grades experience problems with the different aspects of number sense (Reys & Yang, 1998; Van den Heuvel-Paanhuizen, 1996, 2001; Yang, 2005; Verschaffel, Greer & DeCorte, 2007).

McIntosh & Dole (2000) have found that students can calculate accurately, mentally, without understanding, that is, high performance in mental calculation can be achieved without accompanying number sense. It is likely that these students, after much practice, can effectively use a strategy mechanically. Ot McIntosh et al. (1992) claim that high ability in written calculations is not necessarily accompanied by number sense. That is that, a student or an adult can find mechanically the correct answer for an operation without understanding the meaning of the numbers or the operation.

Caney and Watson (2003) conducted a study with 24 students, from grades 3-10, in order to record the strategies they used in performing mental calculations with fractions, decimals and percents. Despite the fact that the sample of students was relatively small, many mental strategies were exhibited in their effort to answer the problems. Many strategies used in solving part-whole number problems seem to coincide with those recorded in whole numbers. Some strategies employed with whole numbers, were not observed in operations with rational numbers. New strategies that were implemented when working with part-whole type numbers have also been recorded.

Callingham and Watson (2004) conducted a study (with a sample of 5,535 students) in order to describe a developmental scale of students' performance in
mental calculations with fractions, decimals and percents. The six levels of developmental scale seem to indicate an increasing understanding of the structure of part-whole numbers and the application of number knowledge gained in whole number contexts (factors, multiples, and place value).

2.2. Number sense or rule-based strategies for rational numbers

Strategies used by students in solving problems involving operations with rational numbers have been documented by several studies. These strategies are divided into conceptual and procedural (Clarke, & Roche, 2009; Post, Cramer, Behr, Lesh, & Harel, 1993; Yang, Reys, & Reys, 2009). Yang et al. (2009) refer to rule-based strategies and number sense-based strategies. In this study we adopt the terms number sense-based and rule-based strategies. Specifically:

**Number sense strategies:** These strategies are not taught in school, but they arise from the students’ ability to deal with rational numbers holistically based on conceptual characteristics of number sense. In number sense strategies include:

a. **Residual thinking** (Post and Cramer, 1987). When comparing \( \frac{3}{4} \) and \( \frac{7}{9} \) a student may think that \( \frac{3}{4} \) has a residual of \( \frac{1}{4} \) or \( \frac{2}{8} \). Consequently the residual for \( \frac{7}{9} \) (\( \frac{2}{9} \)) is less than the residual for \( \frac{3}{4} \) (\( \frac{2}{8} \)). The fraction with the smaller residual is the bigger fraction.

b. **Benchmarking (or transitive)** (Post et al. 1986). When benchmarking, a student may compare a fraction to another well known fraction, such as \( \frac{1}{2} \), or to a whole number (0 or 1). For example, when comparing \( \frac{5}{8} \) and \( \frac{2}{7} \), \( \frac{5}{8} \) is bigger than \( \frac{1}{2} \), and \( \frac{2}{7} \) is smaller than \( \frac{1}{2} \), therefore \( \frac{5}{8} \) is bigger.

c. **Mental picture**. For example, in subtraction \( \frac{3}{4} - \frac{1}{2} \), a student divides an imagined picture of rectangle into 4 parts (Caney and Watson, 2003) and

d. **Converting a fraction or percentage to a decimal.**

**Rule-based strategies:** When using rule-based strategies students are based on memorized rules which are not necessarily linked to deep conceptual understanding. Rule-based strategies include: finding equivalent fractions with a common denominator (for adding, subtracting or comparing fractions), cross-multiplying fractions, applying memorized rules as: “In order to divide two fractions, copy the first fraction, invert and multiply the second fraction.”

3. **METHOD**

3.1. **Sample**

The sample consisted of 462 fifth and sixth grade students who participated in the sixth competition of “Nature and Life Mathematics”, conducted in six towns of Western Macedonia, Greece. 290 students were fifth graders and 172 were sixth graders. Students who took part in this competition were not selected as their participation in the competition was completely voluntary. Their positive attitude
towards mathematics was perhaps their most distinctive feature. Although the participation in the competition is voluntary, students’ performance in mathematics is expected to be higher than the mean performance.

Finally, the students were not taught mental calculation strategies with rational numbers. Thus, the number sense strategies they used were spontaneous and self-developed.

3.2. Procedure

The competition took place out of school hours. The examination lasted one hour and was conducted in writing while the worksheet consisted of two mental calculation tasks and three word problems. In each mental calculation task, the students were asked to provide two modes of solution and to describe how they thought in writing.

2.3. The tasks

Every student of fifth and sixth grade had to answer five tasks, from which both refer to mental calculations with rational numbers and the other three tasks were word problems.

The tasks on mental calculations with rational numbers are listed below:

5th grade
Q51: I calculate with my mind: 1 - ¼. I use two ways to answer. Every time I write the way I thought.
Q52: I calculate with my mind: 1/2:1/4. I use two ways to answer. Every time I write the way I thought.

6th grade
Q61: I compare the fractions 3/7 and 5/8. Which is bigger? I use two ways to answer. Every time I write the way I thought.
Q62: I find the 90% of 40. I use two ways to answer. Every time I write the way I thought.

The three problems for 5th grade were the following:
1. Easter excursion: In an Easter excursion involved 96 people, men, women and children. Men and women were totally 64. Women and children were totally 65. How many men took part in in the excursion, how many women and how many children?
2. The sculptor: Mr. Nick, the sculptor, bought four marble slabs. Each of them had a length of 2.5 meters. How many slabs of one meter can be cut from them?
3. The square: Peter argues that a shape is square when it has four sides which are pair wise parallel. Do you agree or disagree with Peter? Why? You can use shapes in your answer.

The three problems for 6th grade students were the following:
1. **Home distance:** Elena and Niki go to the same school. Elena lives within a distance of 17 km from the school and Niki’s home is located 8 kilometers away from the school. How many kilometers do they live away from each other?

2. **Kittens:** A cat has 6 kittens: one black, one white, one beige, one white-black, one white-beige and one black-beige. Maria chose three so that randomly two of them have at least one common color. How many different choices can we have?

3. **Pyramid:**
The number of cubes placed on each level of the pyramid depends on a rule.
The table shows the number of cubes at the first 3 levels.

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>CUBES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1o</td>
<td>1</td>
</tr>
<tr>
<td>2o</td>
<td>4</td>
</tr>
<tr>
<td>3o</td>
<td>9</td>
</tr>
<tr>
<td>4o</td>
<td>?</td>
</tr>
</tbody>
</table>

a) Find out how many cubes are there at the 4th level of the pyramid. Write all your thinking process.

b) How many cubes will there be at the 10th level of the previous pyramid? How do you know it? Write all your thinking. Write the rule by which you can find cubes for any level.

c) Is the rule that you found always true? Are you sure that this rule applies no matter how big you make the pyramid?

I am sure [ ]
I’m not sure [ ]

### 3. RESULTS

#### 3.1. Performance

In each task, the students were asked to provide the solution in two ways. The table below presents data on the students’ performance:

<table>
<thead>
<tr>
<th></th>
<th>Q51:1/4</th>
<th>Q52:3/4:1/4</th>
<th>Q61:3/7 &amp; 5/8</th>
<th>Q62:90% of 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two correct answers</td>
<td>65 (22.5%)</td>
<td>52 (18%)</td>
<td>67 (39%)</td>
<td>82 (47.5%)</td>
</tr>
<tr>
<td>Only one correct answer</td>
<td>113 (39%)</td>
<td>80 (27.5%)</td>
<td>57 (33%)</td>
<td>56 (32.5%)</td>
</tr>
<tr>
<td>A correct and a wrong answer</td>
<td>25 (8.5%)</td>
<td>48 (16.5%)</td>
<td>15 (9%)</td>
<td>4 (2.5%)</td>
</tr>
<tr>
<td>Two wrong answers</td>
<td>28 (9.5%)</td>
<td>36 (12.5%)</td>
<td>10 (6%)</td>
<td>4 (2.5%)</td>
</tr>
<tr>
<td>A wrong answer</td>
<td>46 (16%)</td>
<td>66 (22.5%)</td>
<td>19 (11%)</td>
<td>17 (10%)</td>
</tr>
<tr>
<td>No answer</td>
<td>13 (4.5%)</td>
<td>8 (3%)</td>
<td>4 (2.5%)</td>
<td>9 (5%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>290 (100%)</strong></td>
<td><strong>290 (100%)</strong></td>
<td><strong>172 (100%)</strong></td>
<td><strong>172 (100%)</strong></td>
</tr>
</tbody>
</table>
As shown in table 1, few students were able to calculate the operations in two ways. In the 5th grade their success was 22.5% and 18%, while in the 6th grade it was 39% and 47.5% respectively. Many students gave only one correct answer (39% and 27.5% of the 5th graders and 33% and 32.5% of the 6th graders). Several students failed to calculate the operations and gave one or two wrong answers or no answer at all (30% and 38% of the 5th graders – 19.5% and 17.5% of the 6th graders).

3.2. Strategies

Table 2 shows the percentages of students who use each strategy:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule-based strategies</td>
<td>180 (67%)</td>
<td>185 (80.5%)</td>
<td>122 (58%)</td>
<td>140 (62.5%)</td>
</tr>
<tr>
<td>Number sense strategies</td>
<td>89 (33%)</td>
<td>45 (19.5%)</td>
<td>88 (42%)</td>
<td>84 (37.5%)</td>
</tr>
<tr>
<td>The number sense strategies</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Converting a Fraction or a Percent to a Decimal</td>
<td>82 (30.5%)</td>
<td>38 (16.5%)</td>
<td>56 (26.5%)</td>
<td>42 (18.75%)</td>
</tr>
<tr>
<td>Mental picture</td>
<td>7 (2.5%)</td>
<td>18 (8.5%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmarks to ½ or 10%</td>
<td>7 (3%)</td>
<td>7 (3.5%)</td>
<td>19 (8.5%)</td>
<td></td>
</tr>
<tr>
<td>Residual thinking</td>
<td>7 (3.5%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduction in unit</td>
<td>23 (10.3%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Below are a few examples of these strategies:

*Rule – based Strategies.* For Q52: \(1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}\). For Q52:½:1/4: \(\frac{1}{2} = 4/1 = 2/4 = 1/2 = 1\). For Q61: The students have to compare 3/7 and 5/8. 3/7 = 24/56 and 5/8 = 35/56. So, 5/8 > 3/7. For Q62:90% of 40: 90/100•40 = 3600/100 = 36.

*Mental picture.* For Q51:1-1/4: “I see 1 as an entire pizza or a clock with four quarters. I remove the 1/4 and ¾ is left”. For Q52: 1/4 fits twice into ½.

*Benchmarks to one half.* For Q61:3/7&5/8: 5/8 is bigger than 1/2. 3/7 is smaller than 1/2. So 5/8 > 3/7.

*Residual thinking.* For Q61:3/7&5/8: 3/7 is 4/7 away from being a whole (7/7). 5/8 is 3/8 away from being a whole (8/8). Because 4/7>3/8, we have 5/8 > 3/7.

*Reduction in unit.* For Q62:90% of 40: 1% of 40 is 40•0,4 = 0,4. So: 90•0,4 = 36.

As can be seen in table 2, the majority of the students fled in the implementation of the rule – based strategies to find the result. Specifically, the percentage of the students who chose the rule – based strategies as a first or second option goes up to 67% in the subtraction 1-1/4 in 5th grade, 80.5% in the division 1/2+1/4 in 5th grade,
58% in comparing fractions (3/7 & 5/8) in 6th grade and 62.5% in finding the percentage 90% of 40 in 6th grade.

This behavior, namely the widespread use of rule-based strategies by the students, may be justified by the “didactic contract” (Brousseau, 1984), which is formed in classrooms where students are encouraged to use only one method (rule of operation).

The percentage of students who chose to convert fraction or decimal to percentage ranges from 16.5% to 30.5%. Conceptual strategies, as using mental representations, using the 1/2 as a reference point, and residual thinking, seem to be significantly less used.

**Number sense strategies**

As mentioned previously, the students were asked to give two solutions to each task. Table 3 below lists the types of strategy (number sense or rule-based) which the students, who were able to give two correct answers, chose and the selections of students who gave only one correct answer. As shown in Table 3, the vast majority of students who gave two ways of solution, chose one rule-based and one number sense strategy. Regarding the subtraction of fractions, this percentage reached 98.5%. It is interesting to note that the vast majority of pupils who were able to give only one correct solution used a rule-based strategy.

**Table 3: Strategies use by students who gave only one or two correct answers**

<table>
<thead>
<tr>
<th>Use of strategies</th>
<th>Q51: 1-1/4</th>
<th>Q52: ½÷1/4</th>
<th>Q61: 3/7&amp;5/8</th>
<th>Q62: 90% of 40</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategies are used by Students who gave two correct answers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 rule-based and 1 number sense</td>
<td>64 (98.5)</td>
<td>33 (62.5%)</td>
<td>61 (91%)</td>
<td>57 (69.5%)</td>
</tr>
<tr>
<td>2 rule-based</td>
<td>0 (0%)</td>
<td>18 (34%)</td>
<td>0 (0%)</td>
<td>19 (23%)</td>
</tr>
<tr>
<td>2 number sense</td>
<td>1 (1.5%)</td>
<td>2 (3.5%)</td>
<td>6 (9%)</td>
<td>6 (7.5%)</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>65 (100%)</td>
<td>53 (100%)</td>
<td>67 (100%)</td>
<td>82 (100%)</td>
</tr>
<tr>
<td><strong>Strategies are used by students who gave only one correct answer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rule-based</td>
<td>99 (87.5%)</td>
<td>74 (95%)</td>
<td>43 (75.5%)</td>
<td>40 (71.5%)</td>
</tr>
<tr>
<td>Number sense</td>
<td>14 (12.5%)</td>
<td>4 (5%)</td>
<td>14 (24.5%)</td>
<td>16 (28.5%)</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>113 (100%)</td>
<td>78 (100%)</td>
<td>57 (100%)</td>
<td>56 (100%)</td>
</tr>
</tbody>
</table>

**3.3. Errors**

The following table categorizes the students’ errors. As shown, most errors are due to the students’ inability to implement the rule of operation correctly. Also, many of the students made errors, trying to implement a rule which corresponds to another arithmetic operation. For example, in subtraction of fractions (1-1/4), many of the 5th grade students inversed the numerator and the denominator of the second fraction and multiplied (influence of the rule of dividing fractions). In division of fractions (1/2÷1/4) some 5th grade students, found a common denominator, divided the...
numerator and kept the same denominator (clear influence of the rule of adding and subtracting fractions). Finally, some students in the 6th grade, when comparing two fractions (3/7 & 5/8) focused on the size of the numerator or the denominator only e.g. “between fractions 3/7 and 5/8, 5/8 is larger, because its numerator is larger or 3/7 is larger because its denominator is smaller”.

Table 4: Classification of errors

<table>
<thead>
<tr>
<th>Type of error</th>
<th>Q51:1-1/4</th>
<th>Q52:⅓×1/4</th>
<th>Q61: 3/7&amp;5/8</th>
<th>Q62: 90%of 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misapplication of the rule of operation</td>
<td>68 (23.5%)</td>
<td>91 (31.5%)</td>
<td>12 (7%)</td>
<td>16 (9.5%)</td>
</tr>
<tr>
<td>Implementation of the rule of another operation</td>
<td>29 (10%)</td>
<td>48 (16.5%)</td>
<td>13 (7.5%)</td>
<td>6 (3.5%)</td>
</tr>
<tr>
<td>Focusing on the numerator or on the denominator only</td>
<td></td>
<td></td>
<td>19 (11%)</td>
<td></td>
</tr>
<tr>
<td>Other errors</td>
<td>23 (8%)</td>
<td>45 (15.5%)</td>
<td>3 (1.5%)</td>
<td>9 (5%)</td>
</tr>
</tbody>
</table>

3.4. Correlation between strategy selection and students’ problem solving ability

In order to determine if strategy selection (number sense or rule-based) is associated with the students’ problem solving ability, two groups of students were formed. The first group included students who used only number sense strategies and students who chose a rule-based and a number sense strategy, since as stated earlier, the students were asked to solve each exercise in two ways. In the second group students who chose rule-based strategies only were included. Thereafter, we compared the performance of the two groups in solving three word problems which were included on the worksheet of the competition. Student’s scores in problem solving ranged from 0 to 3. We performed t-tests and found significant differences between two groups of students. Table 5 below presents the results of this comparison.

Table 5: Correlation between strategy selection and students’ problem solving ability

<table>
<thead>
<tr>
<th>Question</th>
<th>Number sense Mean</th>
<th>Number sense SD</th>
<th>Rule-based Mean</th>
<th>Rule-based SD</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q51:1-1/4</td>
<td>1.54</td>
<td>0.82</td>
<td>0.99</td>
<td>0.68</td>
<td>5.124</td>
<td>p&lt;0.005</td>
</tr>
<tr>
<td>Q52:⅓×1/4</td>
<td>1.61</td>
<td>0.72</td>
<td>1.12</td>
<td>0.80</td>
<td>3.546</td>
<td>p&lt;0.005</td>
</tr>
<tr>
<td>Q61:3/7&amp;5/8</td>
<td>1.48</td>
<td>0.52</td>
<td>1.04</td>
<td>0.54</td>
<td>4.908</td>
<td>p&lt;0.005</td>
</tr>
<tr>
<td>Q62:90%of 40</td>
<td>1.51</td>
<td>0.50</td>
<td>1.05</td>
<td>0.59</td>
<td>4.976</td>
<td>p&lt;0.005</td>
</tr>
</tbody>
</table>

As shown, the performance of the students who chose number sense strategies is significantly better than students who used rule-based strategies only. Probably the students’ ability to interpret the operations conceptually and see the rational
numbers holistically is a useful skill in problem solving that enhances their performance in this area.

4. CONCLUSIONS

The results of this research show that Greek students in fifth and sixth grade have very weak number sense, concerning fractions and percents. They use mostly rule-based strategies that means they use memorized rules to operate with fractions and percents. In addition, classification of errors shows that the students’ errors are due to the fact that the students have not developed intuitive knowledge and understanding concerning rational numbers, but they make errors while trying to implement a written algorithm without understanding its meaning and function. The majority of students, whereas asked to give two ways of calculation, could only give a strategy which, in most cases, was a rule-based strategy. These results are strikingly similar to the approaches used by many middle grade students in Australia, Kuwait, Sweden, United States and Taiwan. For example, Yang (2005) assesses the number sense of 6th-graders in Taiwan with a series of questions, related to whole and decimal numbers. Results indicated that, regardless of performance level, very few number sense strategies (e.g. using benchmarks, estimation or numbers of magnitude) were used. The evidence also revealed that Taiwanese students tended to apply rule-based methods and standard written algorithms to explain their reasoning. The study of Markovits & Sowder (1994) found that about 42% of seventh graders tried to use written methods (finding the common denominator or changing the fractions to decimals) when compared 5/7 and 5/9. About 25% of them didn’t know how to solve it or gave an incorrect answer. The study of Reys & Yang (1998) and Yang & Reys (2002) also found that there were a high percentage of sixth and eighth graders could not meaningfully compare fractions. This is due to they are lacking in number sense. These results are not surprising since, as mentioned above, the mental calculations with rational numbers are not included in the curricula and textbooks which give emphasis on written algorithms. This fact is reflected in the opinions and attitudes of teachers.

Moreover, the data showed that the students, who demonstrate a kind of flexibility in choosing mental computation strategy and use number sense strategies, which reveal a deeper understanding of the rational numbers concept, have better performance in solving mathematical problems compared to students who use rule-based strategies only. Similarly, Louange, & Bana, (2010) in their research on year 7 students find a strong correlation between number sense and their problem-solving ability. Finally, Wai and Kheong (1998) find significant correlation between problem-solving ability and mental calculation ability of primary 5th grade pupils in Singapore.

Wai and Kheong (1998) examined the correlation between problem-solving ability, mental calculation ability and estimation ability of primary 5th grade pupils in Singapore. Results from quantitative analyses showed that the scores of problem solving, mental calculation and estimation were significantly correlated.
Limitations of the study

There were several limitations in this research, among them the small number of operations presented to the pupils of both 5th and 6th grade. Another limitation was the lack of a realistic context for the operations suggested. We could assume that if the operations had been included within a realistic context, the students would have employed different strategies.

Educational implications

The results of the research suggest that the teaching of rational numbers in a way that would be more comprehensible would be beneficial for the Greek students. The use of mental calculations and estimation in the operations with rational numbers and the use of various strategies can help to the improvement of the teaching of rational numbers. Understanding the significance of the operations is most likely to reduce students' errors, which are caused by incorrect and mechanical, -without understanding- implementation of algorithms. It would be interesting to investigate the effects of an experimental teaching intervention with a focus on rational numbers, emphasizing the understanding, the use of mental calculation and the flexibility in operations on students' behavior.

REFERENCES


**BRIEF BIOGRAPHIES**

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