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EDITOR'S INTRODUCTORY NOTE

INTRODUCTION TO SPECIAL ISSUE

Behaviour of students, teachers and future teachers in mental calculation and estimation

We are happy to present the first Special Issue of our new journal “MENON: Journal for Educational Research” which was introduced in 2012. Research in Mathematics Education is a significant area of educational research, which is included in the topics of this journal.

“Behaviour of students, teachers and future teachers in mental calculation and estimation” has been chosen as the subject for this special issue on the ground of a number of reasons which are presented below.

Over the past decades, many studies have been conducted in the field of mental calculations and estimation and more precisely in relation to the definition of these concepts, the identification of the strategies used by various age groups, the relationship with other concepts, such as number sense, the procedural and conceptual understanding among others.

Many educational systems have updated the teaching of numbers and operations in mathematics, incorporating mental calculations and estimations in their elementary and middle education curricula.

Nowadays, it is considered timely to conduct research in the implementation of the teaching of mental calculation and estimation with whole and rational numbers as well as the recording of students' behaviour and the training of pre-service and in-service teachers in these concepts.

During the last decade, researches on mental computation and estimation with rational numbers has been conducted in the Laboratory of “Nature and Life Mathematic” at the University of Western Macedonia, some of which are presented in this issue.

Most of the papers included in this issue, refer to mental calculations and estimations with rational numbers, a topic that is not very common in the literature and covers a wide age range including elementary school students, adults, as well as pre- and in-service teachers. The researches are presented according to the age range of the participants.

In their study Greet, Bert, Torbeyns, Ghesquière and Verschaffel distinguish between two types of strategies for subtraction: (1) direct subtraction, and (2) subtraction by addition, and provide an overview of the results of 5 related studies using non-verbal methods to investigate the flexible use of these strategies in both single- and multi-digit subtraction. Adults, students and
elementary school students with mathematical learning disabilities have participated in this research.

- **Anestakis and Lemonidis** in their study, investigate the computational estimation ability of adult learners and implement a teaching intervention about computational estimation in a Junior High School for Adults. They suggest incorporating computational estimation into Second Chance Schools and into adult numeracy teaching practices in general.

- The two papers of Lemonidis, Nolka, Nikolantonakis and Lemonidis, Kaiafa examine the behaviour of 5th and 6th grade students in computational estimation and in mental calculations with rational numbers, respectively. In these studies, the relation between students’ performance in computational estimation and mental calculations with rational number and problem solving ability are also examined.

Four studies on this issue, refer to the prospective elementary teachers’ behaviour in mental calculation and estimation.

- Anestakis and Desli examined 113 prospective primary school teachers’ views of computational estimation and its teaching in primary school. Results revealed that the majority of prospective teachers identified the importance of computational estimation for both daily life and school.

- In their research Kourkoulos and Chalepaki interviewed and examined through a test 69 pre-service teachers aiming to investigate the factors that contribute to their computational estimation ability. They found five factors that contribute to computational estimation, such as the mathematical background and the attitude towards mathematics.

- Lemonidis, Tsakiridou, Panou and Griva used interviews to examine the knowledge and the strategy use of 50 pre-service teachers in multiplication tables and their mental flexibility in two-digit multiplications by using the method choice / no-choice by Lemaire & Siegler, (1995). The results showed that prospective teachers are not flexible in two-digit multiplications and their flexibility to mentally calculate two-digit multiplications is associated with their knowledge in prep and their prep response time.

- Koleza and Koleli have used a test to study the mental computations and estimation strategies of 87 pre-service teachers. The data revealed that the prospective teachers’ number sense concerning rational numbers, basic concepts of the decimal system and elementary numerical properties was very weak.

- Lemonidis, Mouratoglou and Pnevmatikos studied 80 in-service teachers’ performance and strategies in computational estimation and individual
differences concerning their age and work experience, their attitude towards mathematics and their prior performance in mathematics during high school years.

- The last paper of Lemonidis, Kermeli and Palaigeorgiou propose a teaching intervention to sixth grade students in order to promote understanding and enrich their conceptual strategy repertoire to carry out mental calculations with rational numbers. At the same time, three teachers’ attitudes towards teaching mental computation with rational numbers, were examined.

Finally, I would like to thank all the researchers from Belgium and Greece who contributed with their papers in this thematic issue, the colleagues from the laboratory of "Nature and Life Mathematics", the reviewers of the papers and Elias Indos for the organizational and technical support in the journal.

The Editor of the first Special Issue of “MENON: Journal for Educational Research”

Charalampos Lemonidis
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PROSPECTIVE TEACHER’S EFFICIENCY AND FLEXIBILITY IN PREP AND MENTAL CALCULATION OF TWO-DIGIT MULTIPLICATIONS

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ABSTRACT

Students’ and adults’ knowledge of prep (Campbell & Xue, 2001, LeFevre, & Liu, 1997, LeFevre et al., 1996, Metcalfe & Campbell, 2008) has been studied extensively. However, a few studies have analyzed students’ strategies in mental calculations of multi digit multiplications (Baek Jae-Meen, 1998; Heirdsfield, A. et al. 1999; Murray, H., et al. 1994; Lemonidis, 2013). In addition, there are no studies analyzing adults’ strategies or examining their mental flexibility in two-digit multiplications, as well as comparing this flexibility with their knowledge of multiplication tables. The main research questions in this paper are (a) which strategies are used by prospective teachers, (b) how flexible they are in mental calculations of two-digit multiplications and (c) how this flexibility is associated with their knowledge of the multiplication tables. Their flexibility in strategies employed to multiply two-digit numbers was examined by using the method choice / no-choice by Lemaire & Siegler, (1995). The results showed that prospective teachers are not flexible in two-digit multiplications and they mostly use the written algorithms mentally in order to calculate. Moreover, their flexibility to mentally calculate two-digit multiplications is associated with their knowledge in prep and their prep response time.

Keywords: Prospective teacher’s knowledge, mental multiplication, knowledge in prep, mental multiplication of two-digit number, flexibility in multiplication.
1. INTRODUCTION

Many studies have been conducted examining the prospective primary school teachers’ knowledge in mathematics. In many of these studies it was indicated deficiency in mathematical content knowledge of prospective teachers (e.g. Ball, 1991; Ball, et al. 2008; Lampert, 1986; Ma, 1999). In this paper, a total of 50 Greek students- prospective teachers- was examined by correlating their skills in mental calculations of simple-digit numbers (multiplication tables) and two-digit multiplications.

The multiplication tables is a basic knowledge that is taught for the first time in the second and third grade of elementary school in Greece and it is considered as important and necessary knowledge, because more complex mental and written calculations with multiplications and divisions are based on it. In the recent decades, mental calculations have been a key curricular content on a global basis (e.g. in England (DfEE, 1999, DfES 2007), in the USA (NCTM, 1989, 2000), in the Netherlands, the Dutch Specimen of a National Program for Primary Mathematics (Treffers & De Moor, 1990), in Australia, the National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) etc. In Greece the mental calculations emerge in the Cross Curriculum Framework (Δ.Ε.Π.Π.Σ, 2003). Students should, among the other maths’ operations, be able to perform mental two-digit multiplications. Therefore, in this paper we examined students as adults and especially as prospective teachers, who will be invited to teach these contents, as well as we examined and analyzed their content knowledge. We desided on examining two-digit multiplications because multi-digit multiplications are bound to the use of the algorithm, while two digit ones are closely related to metal mental strategies.

1.1 Research on the multiplication tables in adults

Through the years, the study on the development of prep strategies showed that procedural strategies have been gradually replaced by immediate recall from memory. This strategy appears during the early school years and develops gradually over the next years by storing in memory more and more numerical facts (Koshmider & Ashcraft, 1991; Siegler, 1988; Siegler & Shrager, 1984). When reaching tertiary education, most of the key numerical facts have probably been dealt with so often that their recall from memory is the dominant strategy (Campbell & Xue, 2001; LeFevre, & Liu, 1997; LeFevre et al., 1996).

However, studies related to adults’ solving simple arithmetic multiplication problems have shown that prior experience does not necessarily result into the exclusive strategy use of recalling numerical facts. LeFevre et al. (1996) found that adults use immediate recall in 80% of the tests, but also use repeated addition, rhythmic counting and product construction. The following three factors, identified in literature, seem to have a significant influence on the answers given by students and adults in prep:
a) **Problem-size effect.** It is easier for both students and adults to calculate products with small numbers. As the numbers of the products grow, the difficulty and the mistakes of students or adults increase (Campbell & Graham, 1985; LeFevre et al, 1996). The response time is greater for products with larger numbers (Campbell & Graham, 1985; LeFevre et al, 1996).

b) **Ties effect.** The products containing doubles (i.e. two factors are the same, e.g. 7x7) are easier for students and adults than products with corresponding numbers, which are dissimilar, e.g. 7x6.

c) **Effects of 5-operand problems.** Multiplying with one of the factors being number 5, for example 5x8, is easier for students and adults and is solved faster than other products of comparable size, as for example 6x7 (Campbell, 1994; Campbell & Graham, 1985; LeFevre et al, 1996).

Based on these data from research, we separated the 12 products of the multiplication tables for the needs of the current study into two groups: a) products with small and easy numbers, b) products with large numbers (see 2.3).

### 1.2 Strategies employed in multi-digit multiplications

Although there are several studies examining the strategies of students and adults in single-digit multiplications (prep), there has been a limited number of studies regarding the strategies that students use in two-digit or multi-digit multiplications, (Baek Jae-Meen, 1998; Heirdsfield, et al., 1999; Murray et al., 1994; Lemonidis, 2013). Based on the existing literature, Lemonidis (2013, 258-261) suggested the following classification of strategies for multi-digit multiplications:

1. **Direct Modeling.** They model the problem with objects or a drawing, and they count the total number of objects, the number of groups or the number of objects in each group.

2. **Counting Strategies.** They count the numbers of the product by using all numbering forms, by skipping forward, by repeated addition and by employing doubling strategies, e.g. 4x15: 15, 30, 45, 60, or 8x25: 2x25 = 50, 50 +50 = 100, 100 +100 = 200.

3. **Direct retrieval.** They directly recall from their memory a known multiplication numerical fact or a production of a numerical fact, e.g. 6x11 = 66, 4x12 = 48.

4. **Partitioning number strategies.** They partition one or both terms of the operation in smaller numbers, so that they can multiply them more easily.
   4.1 Partitioning a number based on its positional value. They partitioning one number based on the numerical system positional value and multiply the parts with the other number, e.g. 8x25 = 8x (20 +5) = (8x20) + (8x5) = 160 +40 = 200.
   4.2 Partitioning both numbers based on their positional value. They partition the multiplier and the multiplicand numbers based on the positional value, e.g. 11x18 = (10 +1) x (10 +8) = (10x10) + (10x8) + (1x10) + (1x8).
4.3 Partitioning a number into nondecade numbers. They partition the multiplier or the multiplicand, not based on the positional value of the numbering system, e.g. $8 \times 25 = 2 \times 4 \times 25 = 2 \times 100 = 200$.

5. Holistic or compensating strategy. They regulate one or both terms of the operation, so that the calculation becomes easier, e.g. $8 \times 99 = 8 \times (100 - 1) = 800 - 8 = 792$.

Lemonidis (2013) examined the performance and the strategies used by the fourth grade primary school students in mental multiplications, same operations of multiplication are the same in the current survey. In that study (Lemonidis, 2013, p.268), students were tested before and after a teaching intervention in mental calculations. The 72.5% and 47% of the students succeed in calculating $8 \times 25$ and $8 \times 99$ respectively, while after the intervention success rates increased to 80.5% and 68% respectively. Regarding the use of strategies, students in general used the mental strategy of the written algorithm for two-digit multiplications and they did not use holistic strategy. Specifically, fourth grade students used holistic strategies in $8 \times 25$ and $8 \times 99$ operations by 0% and 14% respectively, before the intervention, while after the intervention rates increased in 32% and 36% respectively.

### 1.3 Flexibility in mental calculations

The method choice / no choice by Lemaire and Siegler, (1995) has been successfully implemented to assess children’s and adults’ strategy choices in diverse mathematical domains, including solving one-step multiplications (Siegler & Lemaire, 1997) and one-step additions and subtractions (Torbeyns, Verschaffel, & Ghesquiere, 2004, 2005), currency conversion (Lemaire & Lecacheur, 2001), computational estimation (Lemaire & Lecacheur, 2002) and numerosity judgment (Luwel, Verschaffel, & Lemaire, 2005).

Ligouras (2012) used the method choice / no choice to examine the flexibility in mental two-digit multiplications of the sixth grade primary school students. In that study, five multiplications were used ($8 \times 25$, $9 \times 21$, $12 \times 18$, $19 \times 30$, $15 \times 49$), which were the same with the present study except for $12 \times 18$ which, in the present study, was 11x18. Moreover, in the present study there was a sixth product, that of $8 \times 99$. The results of Ligouras’ research showed that the average success rate for the choice state was 30.49% while the state for no choice, where they were obliged to use the holistic strategy, success rate drops to 15.96%. The average flexibility rate of students in mental multiplication was very low and reached only 5.92%.

### 1.4 The present study

The main research question was whether future teachers’ knowledge of prep affects their performance and flexibility in mental two-digit multiplications. This paper also aims at answering the following questions: What is the performance, and what
strategies do prospective teachers employ when calculating products of the multiplication tables?
What is the performance, the strategies used and the flexibility in using strategies of prospective teachers concerning mental two-digit multiplications?

2. METHOD

2.1 Participants
The sample of the study consisted of 50 undergraduate students (N = 50) of the Department of Primary Education, University of Western Macedonia Florina. Forty (40) of them were female (80%) and 10 (20%) were male. Their age ranged from 18 to 22 years (M=19.94 years, SD=1.43).

2.2 Procedure
The study was conducted at the Department of Education, University of Western Macedonia, Florina in May and June 2011. Students were interviewed and the questions were presented on a computer, which also measured the response time. The time recording began with the presentation of the question and stopped when the examinee gave his/her first answer. The researcher presented the question to the student, recorded the response and the strategy employed to perform the operation, asking the student to think aloud and express the way s/he thought and observing any obvious external behavior, such as counting on fingers.

2.3. Tasks
Students were asked to solve mentally 12 prep operations and 6 operations of two-digit multiplications. The prep operations were classified into the following two groups:
1. prep with small and easy numbers: 6x6, 3x8, 5x9, 5x7, 4x9, and 7x7.
2. prep with large numbers: 8x9, 7x8, 6x7, 6x9, 9x8, 9x9

In the prep with small and easy numbers, products with small numbers of 3, 4 and 5 were included, as well as the double products of 6x6 and 7x7, which are considered easy. In the prep with large numbers included multiplications by numbers of 6, 7, 8, and 9.

The 6 operations of two-digit multiplications were classified into the following two groups:
1. Multiplication by simple-digit multiplier and two-digit multiplicand: 8x25, 9x21 and 8x99.
2. Multiplications by two-digit multiplicand and multiplier: 11x18, 19x30 and 15x49.
2.4. Choice / no choice method for flexibility

We chose to use the choice / no choice method (Lemaire & Siegler, 1995) to measure the flexibility of students in the six operations of two-digit multiplications.

Students were tested in solving 6 two-digit multiplications by taking part in three interviews. In the first interview (choice), students were asked to calculate each of the 6 multiplications using freely any convenient calculation strategy to solve the operations. It is worth mentioning that the students had as much time as they needed.

In the second interview (no choice 1), students were asked to solve the six multiplication operations, which were presented in a different order from that of the first interview, only by using the holistic strategy. If their response was not extracted by using holistic strategy, it was considered a wrong one (even if it was a correct response). In the third and last interview (no choice 2), students were asked to solve the six operations that were presented in different order, using whatever strategy they wanted except the holistic strategy. If they used the holistic strategy the answer was considered wrong.

Flexibility was measured, with the method choice/ no choice, at six operations with two-digit multiplications for those students who answered correctly. Specifically, students were characterized as flexible in the following two cases:

1. When the strategy used in the first interview was holistic, in the second interview it was also holistic and in the third interview the strategy employed was not holistic. The time they needed to give the correct response was greater in the third interview than in the second one.
2. When the strategy used in the first interview was not holistic, in the second interview it was holistic and in the third interview the strategy employed was not holistic. The time they needed to give the correct response was lower in the third interview than in the second one.

Students were characterized as no flexible in the following two cases:

1. When the strategy employed in the first interview was holistic, in the second interview it was also holistic and in the third interview the strategy used was not holistic. The time needed to give the correct response was greater in the second interview than in the third interview.
2. When the strategy employed in the first interview was not holistic, in the second interview it was holistic and in the third interview it was not holistic.

3. RESULTS

3.1. Accuracy and strategies in multiplication tables

3.1.1. Accuracy in multiplication tables

Two sets of products were presented to prospective teachers: a) products with small numbers and easy to calculate and b) products with large numbers. The performances
for each prep product, the mean time and the standard deviation of the time, as well as the various strategies used in order to find the product are presented in the Table 1.

### Table 1: Effectiveness, mean response time (sec), standard deviation and strategies used for the products in the multiplication tables

<table>
<thead>
<tr>
<th>Operation</th>
<th>Effective (%)</th>
<th>Mean Time</th>
<th>Standard deviation time</th>
<th>Direct retrieval</th>
<th>Derived-fact</th>
<th>Recite all multip. table</th>
</tr>
</thead>
<tbody>
<tr>
<td>9x9</td>
<td>50 (100)</td>
<td>1.14</td>
<td>.756</td>
<td>48 (96)</td>
<td>0</td>
<td>2 (4)</td>
</tr>
<tr>
<td>7x7</td>
<td>48 (96)</td>
<td>1.88</td>
<td>2.454</td>
<td>49 (98)</td>
<td>0</td>
<td>1 (2)</td>
</tr>
<tr>
<td>6x7</td>
<td>48 (96)</td>
<td>2.06</td>
<td>2.985</td>
<td>43 (86)</td>
<td>4 (8)</td>
<td>1 (2)</td>
</tr>
<tr>
<td>5x9</td>
<td>48 (96)</td>
<td>1.44</td>
<td>1.145</td>
<td>44 (88)</td>
<td>2 (4)</td>
<td>2 (4)</td>
</tr>
<tr>
<td>6x6</td>
<td>46 (92)</td>
<td>1.46</td>
<td>1.842</td>
<td>45 (90)</td>
<td>0</td>
<td>1 (2)</td>
</tr>
<tr>
<td>5x7</td>
<td>46 (92)</td>
<td>2.70</td>
<td>4.514</td>
<td>44 (88)</td>
<td>1 (2)</td>
<td>1 (2)</td>
</tr>
<tr>
<td>4x9</td>
<td>46 (92)</td>
<td>3.16</td>
<td>5.474</td>
<td>38 (76)</td>
<td>4 (8)</td>
<td>4 (8)</td>
</tr>
<tr>
<td>9x8</td>
<td>46 (92)</td>
<td>3.08</td>
<td>4.049</td>
<td>43 (86)</td>
<td>2 (4)</td>
<td>1 (2)</td>
</tr>
<tr>
<td>8x9</td>
<td>45 (90)</td>
<td>4.46</td>
<td>8.315</td>
<td>40 (80)</td>
<td>4 (8)</td>
<td>1 (2)</td>
</tr>
<tr>
<td>3x8</td>
<td>43 (86)</td>
<td>2.24</td>
<td>2.591</td>
<td>42 (84)</td>
<td>0</td>
<td>1 (2)</td>
</tr>
<tr>
<td>7x8</td>
<td>41 (82)</td>
<td>3.42</td>
<td>5.990</td>
<td>37 (74)</td>
<td>2 (4)</td>
<td>2 (4)</td>
</tr>
<tr>
<td>6x9</td>
<td>39 (78)</td>
<td>4.34</td>
<td>5.586</td>
<td>32 (64)</td>
<td>3 (6)</td>
<td>5 (10)</td>
</tr>
</tbody>
</table>

In the 12 prep questions the average performance of students is M = 91% and SD = 0.1%. Only 40% of the total number of students answered correctly to all (12) questions, 68% answered correctly to 11 questions and 88% answered correctly to 10 questions. The 12% of the students could not answer the 3 or 4 of the twelve given products in the multiplication tables. The product 6x9 could not be answered by 22% of the students and the product 7x8 could not be solved by 18% of students.

The average success rate in the multiplications by small numbers was 92.33%, with a standard deviation of 1.08%, while the average success rate in multiplications by large numbers was 89.67%, with a standard deviation of 1.38%. No statistically significant differences were recorded between the success rate of multiplications with large and small numbers (t = 1.184, df = 49, p = .242). Therefore, for the statistical analysis that follows, the average of the success rates in small and large numbers multiplications will be used as a single variable, which represent students’ success rate in multiplications.

However, the students needed significantly more time to find the products in the multiplication tables with large numbers in relation to the products in the multiplication tables with small numbers (t = 3.438, df = 49, p <.005). Specifically, students needed an average of 2.15 seconds with a standard deviation of 2.15 seconds in order to find the products in the multiplication tables with small numbers. On the other hand, they needed on average 3.08 seconds with a standard deviation of 3.27 seconds for the products in the multiplication tables with large numbers.
3.1.2 Strategies in multiplication tables

The strategies used by the students to find the products of the multiplication tables with both small and large numbers were the following:

a. direct recall of the product from memory, for example 8x9 they responded immediately,

b. derived - fact, used other multiplications facts or subtractions and multiplications facts (e.g. to calculate 8x9, they calculate 8x10 = 80-8) and

c. recitation of all multiplication table. For example, to calculate the product 6x9 they refer to the whole column of the multiplication table 9, 1x9 = 9, 2x9 = 18, 3x6 = 18, ... 6x9 = 54.

The basic strategy of calculation, which is used by the majority of students (64% to 98%) was the direct recall of the product. Very few students used the strategy of derived – fact. The greater percentage of this strategy (8%) appeared in the products: 6x7, 4x9 and 8x9. Similarly, 10% was the greatest percent regarding the strategy "recite all the multiplication table" and appeared in the calculation of the product 6x9, which seemed to be the most difficult. Based on the strategy of direct recall, the products could be classified into those with a very high percentage (from 84% to 98%) and those with a high percentage (from 64% to 82%) of direct recall: In the first category classified the products: 7x7 (98%), 9x9 (96%), 6x6 (90%), 5x7 (88%), 5x9 (88%) and 6x7 (86%), 9x8 (86%) and 3x8 (84%). In the second category classified the products: 8x9 (80%), 4x9 (76%), 7x8 (74%) and 6x9 (64%).

3.2. Flexibility and multiplication tables

In order to examine possible differences in students’ success rate in the multiplication tables according to their level of flexibility, the technique of One Way ANOVA was used with the success rate as the dependent variable and the level of flexibility as the independent variable. The analysis indicated that students’ success rate in the multiplication tables (F(2,47)= 3.60, p < .05) was significantly affected by the students’ flexibility. More specifically, in accordance with the post hoc LSD test (.05 level), students with a medium level of flexibility are performing significantly higher in the multiplication tables (M = 96.43%, SD = 5.4%) compared to students of low (M = 88.10%, SD = 10.7%) and ‘none’ flexibility level (M = 89.39%, SD = 9.70%). One Way ANOVAs were also used in order to examine possible differences in students’ time response in the case of large numbers as well as in the case of small numbers multiplication, with the level of flexibility as the between subjects variable. The analysis indicated that the level of flexibility significantly affects students’ time response in the case of large numbers multiplication (F(2,47)= 4.94, p < .05). The post hoc LSD test (.05 level) indicated that students with a medium level of flexibility needed significantly less time (M = 0.93sec, SD = 0.46sec) to find the result of multiplication tables with large numbers compared to students of low (M = 3.75sec, SD = 3.14sec) or ‘none’ flexibility level (M = 4.03 sec, SD = 3.79 sec). However, the
analysis indicated that students’ flexibility does not seem to affect the time needed for finding the result in the case of the multiplication tables with small numbers (F(2, 47) = 3.13, p = .053). Nevertheless, the more flexible students need less time (M = 0.98sec, SD = 0.81sec) to find the result of the multiplication tables with small numbers compared to students of low (M = 2.54sec, TA = 1.45sec) or zero (M = 2.64sec, SD = 2.80sec) flexibility level (Table 2).

Table 2. Mean time and standard deviation answer time (sec) in the multiplication tables.

<table>
<thead>
<tr>
<th>Multiplication tables Flexibility</th>
<th>Small numbers</th>
<th></th>
<th>Large numbers</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean time</td>
<td>Standard Deviation</td>
<td>Mean time</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Zero</td>
<td>2.64</td>
<td>2.80</td>
<td>4.03</td>
<td>3.79</td>
</tr>
<tr>
<td>low</td>
<td>2.54</td>
<td>1.45</td>
<td>3.75</td>
<td>3.14</td>
</tr>
<tr>
<td>medium</td>
<td>0.98</td>
<td>0.81</td>
<td>0.93</td>
<td>0.46</td>
</tr>
<tr>
<td>Total</td>
<td>2.15</td>
<td>2.15</td>
<td>3.08</td>
<td>3.27</td>
</tr>
</tbody>
</table>

3.3. Accuracy and strategies in mental multiplication of two digit numbers

3.3.1 Accuracy

Table 3. Performance and strategies in two-digit multiplications

<table>
<thead>
<tr>
<th>Operation</th>
<th>Accur. (%)</th>
<th>Accur. (%)</th>
<th>Accur. (%)</th>
<th>Accur. (%)</th>
<th>Accur. (%)</th>
<th>Accur. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x25</td>
<td>40 (80)</td>
<td>21 (42)</td>
<td>-</td>
<td>11 (22)</td>
<td>18 (36)</td>
<td>43 (86)</td>
</tr>
<tr>
<td>9x21</td>
<td>33 (66)</td>
<td>35 (70)</td>
<td>-</td>
<td>8 (16)</td>
<td>10 (20)</td>
<td>41 (82)</td>
</tr>
<tr>
<td>8x99</td>
<td>26 (52)</td>
<td>30 (60)</td>
<td>15 (30)</td>
<td>-</td>
<td>19 (38)</td>
<td>31 (62)</td>
</tr>
<tr>
<td>19x30</td>
<td>31 (62)</td>
<td>21 (42)</td>
<td>10 (20)</td>
<td>16 (32)</td>
<td>25 (50)</td>
<td>37 (74)</td>
</tr>
<tr>
<td>11x18</td>
<td>26 (52)</td>
<td>24 (48)</td>
<td>1 (2)</td>
<td>20 (40)</td>
<td>19 (38)</td>
<td>16 (32)</td>
</tr>
<tr>
<td>15x49</td>
<td>12 (24)</td>
<td>14 (28)</td>
<td>10 (20)</td>
<td>10 (20)</td>
<td>17 (34)</td>
<td>15 (30)</td>
</tr>
</tbody>
</table>

As shown in Table 3 in the first column of the situation choice, students’ performance in mental two-digit multiplications ranged from 24% (15x49) to 80% (8x25).

The success rate in multiplying a two-digit multiplier and single-digit multiplicand (M = 66%, SD = 30%) is significantly higher than the success rate in multiplying two-digit multipliers and two-digit multiplicands (M = 44%, SD = 30%). This difference in student success rates in both cases of multiplications is statistically significant (t = 3.66, df = 49, p < .005). For this reason, the two multiplication cases were studied separately and the results are presented below, separately for each case. Moreover, the time that students needed to find the result of multiplications in case of two-digit multiplicand and multiplier (M = 20.90sec, SD = 8.49sec) is significantly higher (t =
5.538, df = 49, p < .001) compared with that of a single-digit multiplicand and multiplier (M = 12.94sec, SD = 8.18sec). The success rates both in the case of a multiplication by a single-digit multiplier and two-digit multiplicand (t = 5.597, df = 49, p < .001) and in the case of a multiplication by a two-digit multiplier and multiplicand (t = 11.060, df = 49, p < .001) were significantly lower than the students’ success rates in the multiplication tables. However, there was a statistically significant correlation with success in multiplication tables only in the case of multiplication by a two-digit multiplier and multiplicand (Table 4), which is actually low (r = 0.288, p < .05).

**Table 4. Correlation success rates in multiplication tables and two-digit multiplications**

<table>
<thead>
<tr>
<th>Operations</th>
<th>r</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prep &amp; one two – digit number</td>
<td>.038</td>
<td>.796</td>
</tr>
<tr>
<td>Prep &amp; two – digit numbers</td>
<td>.288</td>
<td>.042</td>
</tr>
</tbody>
</table>

### 3.3.2 Strategies

Concerning the strategies used by the students in order to mentally calculate two-digit multiplications, the following can be highlighted. Table 3 presents the percentages of the main strategies used by the students in the first interview, where they freely chose their preferred strategy to calculate (situation choice). The basic strategy that students use is the mental representation of the written algorithm (e.g. in the operation 9x21), they imagine the vertically written algorithm and calculate: 1x9 = 9, 2x9 = 180, vertically add 9 +180 and they find the result 189. A small number of students use a holistic strategy or compensation strategy, e.g. in the operation 9x21 they convert 9 to 10 and calculate 10x21 = 210, 210-21 = 189 or in the operation 8x99 they count 8x100 = 800, 800-8 = 792. In the second, ‘nochoice’ 1 interview, where students had to calculate using a holistic strategy, the rates of using this strategy are low, with the highest being 50% in the operation 19x30, as shown in Table 3. It is therefore proved that this strategy was not known to the students and after being informed about it, they had difficulty to employ it. Some students also used the strategy of portioning a number, where separate two-digit numbers mainly in units and tens (e.g. in the operation 9x21), they partition 21 into 20 and 1 and multiply 9x21 = 9x (20 +1) = 180 +9 = 189.

### 3.4. Flexibility in mental multiplication of two digit numbers

The percentages of flexible students in relation to each operation are presented in Table 5.

**Table 5. Frequencies and percentages of flexible students per operation**

<table>
<thead>
<tr>
<th>Operation</th>
<th>8x25</th>
<th>9x21</th>
<th>8x99</th>
<th>19x30</th>
<th>11x18</th>
<th>15x49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of flexibility (%)</td>
<td>10 (20%)</td>
<td>8 (16%)</td>
<td>4 (8%)</td>
<td>17 (34%)</td>
<td>6 (12%)</td>
<td>6 (12%)</td>
</tr>
</tbody>
</table>
It is observed that the percentage of students who are flexible in mentally multiplying two-digit numbers is very small (17%). Only in the operation 19x30 flexible students reached one third of the students (34%), while in the rest of the operations the percentage of flexible students is much smaller, with the smallest percentage being 8% for the operation 8x99. Based on students’ flexibility in mental multiplications, their flexibility in all operations was estimated and was categorized on three levels of flexibility: zero, low and medium. Students classified as of zero flexibility were not flexible in any operation of all given and represent 44% of the sample. Students classified as of low level of flexibility were flexible in only one of the six operations and represent 28% of the sample, while students classified as of medium level of flexibility were flexible in 2.6 out of the six operations and represent 28% of the sample. It is worth mentioning that there were no students to be flexible in all operations, while there was only one student who was characterized as flexible in five out of six operations.

3.4.1 Flexibility and multiplications by single-digit multiplier and two-digit multiplicand

Students’ level of flexibility affects significantly their performance in multiplications by a single-digit multiplier and a two-digit multiplicand ($F_{2, 47} = 4.895, p < .005$). According to the LSD test, students with medium level of flexibility show higher success rates ($M = 85.71\%, SD = 17.11\%$) compared to students of low flexibility level ($M = 60.61\%, SD = 30.23\%$) or zero flexibility level ($M = 54.76\%$, $SD = 33.61\%$). Moreover, the time that students need to find the result of multiplications by a single-digit multiplier and two-digit multiplicand is significantly influenced by the students’ flexibility level ($F_{2, 47} = 4.246, p < .05$). The analysis continued with the LSD test, and it was indicated that students of medium level of flexibility needed significantly less time to find the result of these multiplications ($M = 8.19$ sec, $SD = 5.84$ sec), compared to students of low flexibility level ($M = 13.73$ sec, $SD = 7.77$ sec) or zero flexibility ($M = 16.45$ sec, $SD = 9.07$ sec).

3.4.2 Flexibility and multiplications by a two-digit multiplier and a two-digit multiplicand

Students’ flexibility level does not affect their performance in case of multiplications by a two-digit multiplier and a two-digit multiplicand ($F_{2, 47} = 0.466, p = .630$). Thus, students, regardless of their flexibility level, show a comparatively low performance in the case of a two-digit multiplier and multiplicand, with an average success rate of 46% and a standard deviation of 30%. Also, the time students need to find the result of these multiplications, does not seem to be significantly affected by the students’ flexibility level ($F_{2, 47} = 3.117, p = .054$), although students of medium flexibility level need less time ($M = 16.29$ sec, $SD = 6.39$ sec) to find the result, compared to students of low flexibility level ($M = 22.69$ sec, $SD = 5.87$ sec) or zero flexibility level ($M = 22.69$ sec, $SD = 10.10$ sec).
4. DISCUSSION AND CONCLUDING REMARKS

To sum up, the results of this research concerning the knowledge of prospective teachers in multiplication tables and mental calculations with two-digit multiplications are the following: the participants were found to ignore the products of some multiplication tables. The most difficult product being 6x9 could not be calculated by 22% and could not be immediately recalled from memory by 36% of students. Also 22% of students could not calculate two out of the 12 given products. Regarding the strategies used, it was observed that the dominant strategy was that of direct recall from memory which was used by 64% to 98% of the students. Other strategies used alternatively by students at much lower rates are the strategy of derived fact, which was used by 8% of the students and the strategy of recite all multiplication column, which reaches the percentage of 10% for the difficult product 6x9. The products of the multiplication tables with small numbers were revealed to show higher success rate and shorter response time. These results are in accordance with those of by Le Fevre’s et al. study (1996) conducted with introductory psychology students. They reported direct retrieval on approximately 80% of trials, but also reported rules (e.g., anything times 0 is 0), repeated addition (e.g., 2x4 = 4 + 4), number series (e.g., 3x5 = 5, 10, 15), and derived facts (e.g., 6x7 = [6x6] +6).

Concerning the performance and strategies used by students for mental two-digit multiplications we came to the following conclusions. The mental single-digit by two-digit number multiplications presented a significantly higher rate of success than the two-digit by two-digit number multiplications (66% - 44%). Also, single-digit by a two-digit number multiplications present statistically smaller response time by two-digit by two-digit number multiplications. Comparing the results of this study with those of Ligouras’s study (2012) conducted to sixth grade primary school students, who solved almost the same operations, we come to the conclusion that the university students’ success rate in two-digit multiplications (56%) is higher than that of the primary school students (30.49%).

Regarding the strategies used by the students for the calculation of two-digit multiplications in the free situation choice (choice) the dominant strategy was the mental representation of the written algorithm (48.3%). Meanwhile, a certain number of the students also used the strategy of “partitioning number” (34%) and a small percentage (18%) used holistic strategies. The fact that they did not know how to use holistic strategies was also demonstrated by the success rate (36%) in no choice 1 situation, where they had to use holistic strategies. This percentage is, however, larger than that (15.96%) of the primary school students in the Ligouras’s study (2012). The ignorance of holistic strategies is possibly one of the main causes of low flexibility, with an average rate of 17%, that students present regarding two-digit multiplications use of strategies. This flexibility percentage is greater than the very low flexibility rate (5.92%) of the sixth grade school students in similar multiplications in the research conducted by Ligouras (2012).
Concerning the relationship of success in two-digit multiplications and the success in the multiplication tables we observed that the correlation of prep with a two-digit multiplied by a single-digit number is very small and not statistically significant (r = 0.038, p > .05), while the correlation of the prep with multiplications of a two-digit by a two-digit number is statistically significant but there is a low correlation index r (r = 0.288, p < .05). We can therefore assume that for the success in mental two-digit multiplications the knowledge of the multiplication table is not enough. Moreover, students which are efficient in multiplication are not necessarily efficient in mental two-digit multiplications.

The findings also showed that students with medium flexibility in mental calculations can achieve statistically higher success rates at prep and statistically shorter response times in prep with large numbers and marginally shorter response times in prep with small numbers compared to students who are not flexible. So, though it seems that there is no strong relationship between their performance in mental two-digit multiplications with their performance in prep, on the contrary university students with medium flexibility in mental two-digit multiplications show highest success rates and shorter response times. This means medium flexibility students in two-digit multiplications are better at prep and recall it faster from memory than students who are not flexible. Finally, it was revealed that students of medium level of flexibility achieve statistically higher rates and shorter response times in mental single-digit multiplications by a two-digit number. Although students of medium flexibility level do not present statistically higher success rates in mental multiplications of a two-digit by a two-digit, their response time of these students is marginally not statistically smaller than that of students of low or none flexibility.

In relation to the prospective teachers’ skills in mental multiplications we can draw the following conclusions. The majority of the participants seemed not to know the prep and not use direct recall as an exclusive strategy. Le Fevre et al. (1996) also drew these conclusions conducting a study with psychology students.

The majority of the participants did not show a high level flexibility in the strategy use. Also, very low flexibility levels regarding Greek students of the sixth primary school grade were found in Ligouras’s study (2012), for mental multiplications additions and subtractions. Although for the majority of the participants there is no strong correlation between success in prep and two-digit multiplications, students of medium flexibility level achieve statistically higher success rates and shorter response times. In addition, the students of medium flexibility level seemed to tend to have better success rates and response times than students who are not flexible.

**Educational considerations**

Based on the results of this study, we can assume that the participants were not taught in the logic of mental calculations as pupils with the aim to develop flexibility in strategy use. The findings of the study indicated that this lack of flexibility in strategy use, which is evident in prospective teachers’ skills, probably was one of the reason of low success rates at mental two-digit multiplications. In order to
compensate this lack of flexibility related to prospective teachers’ skills, there is an immediate need for delivering training courses integrating theory and practice. For this purpose, it is suggested to think of providing such a training program.

REFERENCES


**BRIEF BIOGRAPHIES**

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