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Positions numeration in any base for future Elementary school teachers in France and Greece: one discussion via Registers and Praxis

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Abstract

The numeration in any base for whole numbers is a mathematical topic which introduces new registers of representation. We reinterpret the question for treatments in the new registers in the language of praxeologies: new praxis is to be built. In this paper, our first objective is to examine the cognitive distinction between treatment and conversion made by Duval for the registers of numeration in base other than ten. Two inquiries was made and analyzed with the Statistical Implicative Analysis (SIA) that has permitted to show two cognitive difficulties for coding numbers into the position numeration in any base that comes from: 1) the base is different from ten and 2) the base is greater than ten. Our second objective concerns the technological potential of polynomial writing. The SIA permits us to show that this register is important for the success of the types of tasks and for the construction of techniques in any base.

Keywords: Position numeration, any base, registers, praxis, elementary school teachers

1. Introduction

The aim of this paper is to compare the training of first degree teachers in mathematics in France and Greece. In this article we present some questions raised by the numeration in any base. It is a theme which can bring out phenomena that can be analyzed by both praxeologies and registers of representation.

To analyze the items on the numeration in any base, we use the relationship, introduced in Vivier (2008), between the Anthropological Theory of Didactics (ATD) of Chevallard (1999, 2002) and the registers of semiotic representation of Duval (1993, 1996, 2006). The ATD provides a framework for linking techniques with the types of tasks by inserting them into a mathematical organization, which is not permitted directly by Duval’s framework. On the other hand, the distinction of techniques between treatment and conversion allows, for non-congruent conversion, to distinguish a cognitive difference pointed by Duval that praxeologies generally do not permit to analyze.

Our frame of analysis seems well adapted for studying the introduction of a new register of representation for which new techniques must be elaborated. We would like to point out that it is not our intention to articulate Chevallard’s and Duval’s frameworks. We would like to take into account semiotic registers within praxis by restriction to the numerical frame.

We have proposed two tests on position numeration in any base which is, restricted to ten base, a central teaching theme in elementary school. After a preliminary study, some questions arose. Our goal is to address these questions and answer them, by a statistical study. The types of tasks proposed in the main test are much more numerous than in the
preliminary test and didactic variables are adjusted for the needs of this study. Data collected through the main test were analyzed through the software CHIC (Gras et al. 2009).

We start by presenting theoretical link between frameworks of Chevallard and Duval. We stress the importance of taking into account praxeologies and registers simultaneously. Then, we describe the context of the studies and teachers’ training systems in both countries. The results are presented in section 5 for total population and section 6 for a comparison of the two populations.

2. Theoretical Framework

In this section we develop the relationship between praxis (Chevallard 1999, 2002) and registers of representation (Duval 1993, 1996, 2006) that we will use in the analysis of the next sections.

Via a distinction of techniques from registers, we would like to preserve a part of students’ cognitive activity revealed by Duval. Besides a better praxeological description, this theoretical work provides, as it is the case in this study, results which are not easily detectable by other experimental means.

2.1 Praxis in ATD

In the Anthropological Theory of Didactics (ATD) the mathematical activity is elaborated around type(s) of tasks and it is appointed by Chevallard (1999) as mathematical organization. Generally, to perform a type of tasks $T$, we have at least one technique $\tau$. Type of tasks and techniques are organized in a $[T, \tau]$ appointed block of know-how or praxis. If no technique is available, the type of tasks $T$ is problematic and requires the production of a technique $\tau$ (Chevallard 1999). To produce and/or justify a technique $\tau$, it is necessary to have a theoretical look at the problem posed by T. Chevallard defines a block of knowledge or logos, made up of technology and theory. Type of tasks, techniques, technology and theory form a praxeology, called a specific praxeology when one considers only one type of tasks.

2.2 Registers of semiotic representation

In his framework, Duval takes another point of view. He starts from signs used in the mathematical work grouped into registers of semiotic representations (Duval 1993, 1995, 1996, 2006). The essential distinction made by Duval consists of the dichotomy between treatment and conversion. Treatment is a semiotic transformation which remains within the same register of representation R. Conversion is a semiotic transformation whose result is expressed in another register.

Duval (1996), stresses the essential cognitive difference between treatment and conversion. Conversion is much more complex and problematic than treatments, especially a non-congruent conversion. However, as Duval stated, two reciprocal conversions are generally not cognitively equivalent.
2.3 Praxis indexed by registers

In ATD, a type of tasks is not always stated with reference to the register(s) of representation in use. However, a student task is always expressed by using semiotic registers. These can significantly influence the cognitive activity of students.

To take this distinction into consideration we index the types of tasks by the registers in which they are expressed. When the type of tasks T is relative to a single register of representation R, we write $T_R$. Similarly, a technique $\tau$ specific to one register R, that is to say a treatment of R, is denoted by $\tau_R$. Thus, we obtain a praxis relative to a register R which we write $[T_R, \tau_R]$ or simply $[T, \tau]_R$ and we call it a R-praxis. Within a single register R we could schematically represent the situation by the figure 1.

Figure 1: praxis in register R

![Diagram of praxis in register R]

We should distinguish between two types of techniques denoted by $\tau_R$. This could be a pure treatment (Duval 1993), meaning one internal semiotic transformation to a register R, or another type of technique which provides one external response to register R. For this second type we can give one example: to determine the parity of a written number in base $a$ there is not necessarily a transformation (if $a$ is even) and especially the result is not expressed in the initial register – this is not a treatment in the strict sense of Duval.

2.4 Conversions

The previous framework is very local and does not permit, by itself, any modelisation of the mathematical activity. For example, a type of tasks can be expressed in several registers which requires the coordination of registers and conversions. To build a link between Chevallard’s and Duval’s point of views, we consider a conversion between two registers $R \rightarrow R'$ as a technique $\tau_{R \rightarrow R'}$. These conversion techniques are, according to Duval, very different in a cognitive point of view from the treatment techniques which are expressed in a single register.

Likewise, if it is clear that a mathematical technique is not always internal to a single register, our analysis framework permits to decompose one mathematical technique as a succession of techniques of one of the two preceding types.

Duval states that the fundamental cognitive functioning of mathematics requires the coordination of at least two registers of representation (Duval 1996, 2006). We reinterpret his remark by using the language of praxeologies. We consider two registers $R$ and $R'$ and two reciprocal conversion techniques $\tau_{R \rightarrow R'}$ and $\tau_{R' \rightarrow R}$. These, as reported by Duval (1996), permit the coordination of registers. But coordination goes further: these conversion techniques allow the coordination of treatments, $\tau_R$ and $\tau_{R'}$, and also the types of tasks, $T_R$ and $T_{R'}$. Finally the conversion techniques allow the coordination of R-praxis, $[T, \tau]_R$ and $[T, \tau]_{R'}$ (figure 2).

---

1 We explain in (Block, Nikolantonakis & Vivier, 2012) why this point of view is restrained to the numerical domain.
But we will see that conversion techniques also allow avoidance strategies: if one wants to solve $T_{R'}$ then the production of a technique $\tau_{R'}$, specific to the register $R'$, could be blocked by the knowledge of $[T,\tau]_R$. It is sufficient to use $\tau_{R'\rightarrow R}$ and then solve $T_R$ by using $\tau_R$ - then possibly use $\tau_{R\rightarrow R'}$ if one wants to go back to the original register $R'$. From the ATD point of view, a conversion technique is a technology: $\tau_{R'\rightarrow R}$ allows to produce a technique for solving $T_{R'}$, by using the R-praxis $[T,\tau]_R$. However, as we will see in this case and in our study, a conversion has to be seen as a technique rather than a technology. The situation is summarized in figure 3 which should be compared with figure 1.

Of course, we could understand the situation with the praxeologies only by specifying the register in the statement of one type of tasks. A technique for solving $T_{R'}$ will be either to work in one register, with $\tau_{R'}$, or to do a conversion, with $\tau_{R'\rightarrow R}$ before doing a treatment in the register $R$, with $\tau_R$. But putting treatment and conversion on equal terms, this modelling masks the essential cognitive difference revealed by Duval.

### 2.5 Necessary restriction to numerical framework

The distinction among techniques described previously permits a conciliation of the two frameworks on the crucial point of conversions.

Firstly, the distinction between treatment and conversion is maintained and by following Duval, we insist on the fundamental different character of the two types of techniques that we note with generic way $\tau_R$ and $\tau_{R'\rightarrow R}$. But this is not sufficient because what we call the technique of conversion should be consistent from the didactic point of view.

In every conversion we find one algorithmic (or algorithmisable) part which corresponds to mathematical work and constitutes the tangible part of conversion (notably from writing traces). Without reducing one conversion to an algorithmic work, it is this algorithmic part which we call conversion technique.
In particular and this is a very important and essential restriction: we can reasonably consider one conversion technique $\tau_{R \to R'}$ only when the passage from $R$ to $R'$ could be described with algorithmic mode or at least when cognitive variables of the situation have very little influence on the development of the algorithm. Duval (1995: 42) points out that we cannot always define conversion rules. This fact put limits on the significance of our proposition.

That is why we restrain the study to numerical frame because generally one conversion between two registers of representation of numbers could be described by an algorithm. It is clear that in our analysis framework, from the specific cognitive character of a conversion we conserve a part which includes notably the conscient choice of a subject to do this conversion. Sometimes the cognitive difference between a conversion and a treatment could be erased totally because only algorithmic component, which is a technique in the sense of ATD, is to the responsibility of the subject. This could be either because we ask explicitly the subject to do this conversion either it is corresponding to a technique which has been worked in the institution for doing some type of tasks.

### 3. Methodology

#### 3.1 Context

In Greece four years of training in pedagogical university departments are fully allocated to primary education against only two in France at the University Institutes of Teachers Training (IUFM).

In Greece, primary education continues through grade 6 included while in France it ends after grade 5. Thus, teachers of first degree in Greece must master to teach more mathematical content than their French colleagues.

However, the two training systems are similar on the subject of the numeration in any base. The courses on the numeration in any base, in the two institutions aim at making students understand the relativity of the base ten while understanding the general mechanisms of the position numeration the following points are addressed and worked upon:

- encoding;
- ordinal aspect;
- conversion between two bases;
- some simple calculations to better understand the mechanism included in the operative techniques.

The numbers of teaching hours on this subject are similar: 9 hours of courses on the subject for Greek students and 6 hours for French students. French and Greek students have similar expertise with the numeration in any base

#### 3.2 Research Methodology

We have proposed two tests on position numeration in any base which took place in the IUFM of Tours, France, and in the pedagogical university of Florina, Greece, in 2009 and 2010. In preliminary test (Annex 1), French and Greek populations (100 Greek students and 26 French students) seem very close. Two hypotheses came out from this test:
Different systems of position numeration could be interpreted as different registers;

- Adjusting didactic variables permits the emergence of the polynomial writing as a full register, with its technological potential, and not as a simple transitional writing for a conversion between an a base and ten base.

The principal objective is the validation of these two hypotheses. By the main test we want to compare the Greek and French populations more deeply by using CHIC software (Gras et al. 2009). Hence, we have extended our efforts in two directions: we increased the size of the French population (total population 334 students, 195 French students and 139 Greek students) in order to provide statistically satisfactory conclusions and studied praxis more extensively with more types of tasks, always in bases other than ten. In our main test we have added three more tasks (Annex 2). The goal is to look for similarities between variables in order to bring out the different character of treatments and conversions but also, by implicative trees, the quasi-implications between variables, mainly those variables relative to the use of polynomial writing register $R_{poly}$.

**4. Discussion concerning the total population**

In this section we validate, for the total population, the two hypotheses cited above coming from the preliminary study. We recall here: (1) different systems of position numeration could be interpreted as different registers; (2) adjusting didactic variables permits the emergence of the polynomial writing as a full register, with its technological potential, and not as a simple transitional writing for a conversion between an a base and ten base.

**4.1 Registers of representation of numbers in base a**

The coding of whole numbers in base $a$ system constitutes a register of representation in the sense of Duval (1993, 1996), denoted by $R_a$. From a mathematical point of view, all these registers are equivalent, and very similar in their functioning. From a cognitive point of view, any variation in $R_a$ induces a variation in $R_b$: the variations of any kind are cognitively relevant. In addition, these conversions are not congruent at all: in general all the digits as well as the number of digits change.

To convert $R_a \rightarrow R_b$ for a number $N$, there are essentially two conversion techniques:

- $\tau_{a,b}^{\text{div}}$, making successive Euclidean divisions by $b$ in $R_a$, starting from the Euclidean division of $N$ by $b$. The successive remainders, by identifying them with the digits in $R_b$, give digits of $N$ in $R_b$ (see figure 4);

**Figure 4:** G12 from the Preliminary test, the technique of reconversion with division

- $\tau_{a,b}^{\text{pd}}$, from polynomial development in $R_a$, converting $a$ and the digits of $a$ in $R_b$, then calculating in $R_b$ (see figures 5a and 5b).
Figure 5a: F9 from the Preliminary test- Polynomial development allows one conversion and then a treatment in $R_{ten}$, the reconversion is explicit, but without visible technique.

Figure 5b: G14 from the Preliminary test, an error in using $\tau_{pd}^{a,ten}$

These conversions require mastering the operating techniques in $R_a$, or $R_b$, which is generally the case only in $R_{ten}$. Thus, we can think reasonably that:

- the conversion $R_{ten} \rightarrow R_b$ is often performed by the conversion technique $\tau^{div}_{ten,b}$;
- the conversion $R_a \rightarrow R_{ten}$ is often performed by the conversion technique using the polynomial development $\tau^{pd}_{a,ten}$;
- the conversion $R_a \rightarrow R_b$ involving two different bases from the base ten is often performed by the double conversion $R_a \rightarrow R_{ten} \rightarrow R_b$ by using the polynomial development $\tau^{pd}_{a,ten}$ and then the Euclidean division $\tau^{div}_{ten,b}$.

In continuation of the preliminary study, many points seem to indicate that different registers are involved. On the one hand, it is clear that signs relative to every register differ since the digits are not the same, even though, when $a>b$ every digit of $R_b$ can be interpreted as a digit of $R_a$. In the other hand, some treatments are different.

In addition, the similarities tree on the total population of main test (figure 6) shows that treatments and conversions are distinct. We find the cognitive distinction between treatment and conversion made by Duval when we have to deal with distinct registers. It should be noted that only statistical analysis permits to find this distinction. In fact, during our preliminary study (Annex 1) we only made the analysis with percentage of error which does not permit distinguishing treatment from conversion.
Thus, if we can always discuss resemblances or differences between registers $R_a$ and $R_b$, we validate statistically our first hypothesis, at least for $R_a$ and $R_{ten}$. In this tree one can see that No Responses (NR) to the 9 items constitute very similar variables which were all regrouped at level 8 (similarity index 1). They concern approximately 20% of students with disparities between items (annex 2). These students were divided into two groups: those who are put off by registers $R_a$ when $a\neq ten$ and those who were bothered by the types of tasks in register $R_a$, that means the $R_a$-praxis. Thus, the registers do not constitute the only interpretation element since it has been combined with praxis. And for this reason we are using the framework presented in section 2.

Separately, the two populations have very close profiles with those evoked previously (see annex 3). Treatments error variables were grouped with « NR » variables, the formed block was not grouped with other variables and notably with conversions error variables.

4.2. One hierarchy of $R$-praxis

The main test proposes 5 types of tasks. Annex 2 shows, by the rate of success (OK) and no response (NR), that these types of tasks are distinct. Moreover, in blocks « errTreat », « errConv », « OK » or « NR » of figure 6, we identify sub-blocks associated with the types of tasks SP and EO.

The implicative graph of the total population gives one hierarchy of the difficulty order of items (Annex 4, figure 11). The quasi-implications of « OK » and « NR » propose a identical hierarchy to the inversion SPa/SPc and to the particular case of SPd. The simplest type of tasks is Class (6% of NR and 82% of OK) which constitutes, with SPd, the only item which can be carried out correctly by application of knowledge of base ten. The most difficult tasks are EO, notably in odd base (EOb : 19% of NR and 51% of OK) and the coding on Eleven base (Elev : 17% of NR and 39% of OK). The difficulty of the item EOb could be explained by the proposed number, which has a big number of digits, and by the parity of base which put in default the usual technique of $R_{ten}$. 

Figure 6: Similarities tree for total population (Fr+Gr) of main test.
The difficulty of the second type of tasks, Elev, is much more surprising to the extent that the coding of numbers in bases other than ten has been worked, even though it is clear that essentially the difficulty lies on the fact that the base is strictly superior to ten and we should code ten as a digit.

We deduce the following hierarchy of R-praxis, where we put in index the characteristics of the base $a$ of register $R_a$:

\[
\text{Elev}_{a \gt \text{ten}} - \text{EO}_{a \text{ odd}} - \times 8_{\text{a ten}} - \text{EO}_{a \text{ even}} - \text{SP}_{\text{a ten}} - \text{Class}_{\text{a ten}}.
\]

### 4.3. Technological potential of polynomial writing register

The similarity tree of the whole population (figure 6) shows a gather together on significant level 18 (index of similarity 1) of three variables «poly» of the three concerned items. This is not surprising since polynomial writing constitutes a very efficient register to treat the types of tasks for which we do not have an institutional technique\(^2\).

Of course, the procedure which consists of doing firstly one conversion in $R_{\text{ten}}$ is always possible, but the size of number is, for EO, a didactic variable which makes conversion difficult. Thus, we can think that some students, on the way for a conversion, stay on the polynomial writing by asking themselves if it is reasonable to calculate this great expression and understand that, for EO$_a$, all terms of sum are evens.

They conclude on the parity of number by proceeding to a treatment in $R_{\text{poly}}$. Some of them enunciate and justify in a correct manner the techniques that are the criterions of parity, according to the parity of the base (see figures 7a and 7b).

**Figure 7a:** F66 from the Main test, production of parity criterion into $R_{\text{eight}}$

![Figure 7a](image1)

**Figure 7b:** F74 from the Main test, production of parity criterion into $R_{\text{seven}}$

![Figure 7b](image2)

We have not thought that the technological potential of $R_{\text{poly}}$ was realized so strongly. Our second hypothesis is validated – independently from the weak percentage of

\(^2\) This is partially true for «$	imes 8$» since we found it frequently in the technique which consists of «sum up a zero» like G61 who wrote: “In the octal system 8 is 10. So $3405 \times 10 = 34050_8$.”
students who used $R_{poly}$ as a technological element (4% for $\times 8$, 11% for EOa and 8% for EOb).

This technological element is not anecdotal as we can observe on the implicative graph for the whole population (Annex 4, figure 12). The three variables « poly » are on the top of the sub-graph containing the variable « OK », that seems to indicate one important variable for the success of the types of tasks.

No Greek student has used $R_{poly}$ register in a way other than transitory register between $R_a$ and $R_{ten}$; $R_{poly}$ register was considered as a stage of conversion. Thus, our second hypothesis was finally validated for the French population only.

5. Comparison of two populations

5.1 Worked types of tasks

The type of tasks SP is effectively worked in the two institutions and was very well succeeded with a difference between S and P. The similarity tree for the French population shows a gather together of all « OK » variables on significative level 44 (index 0.08; Annex 3) with two blocks: one with SP-OK and Class-OK (level 42, index 0.63) and one other with EO-OK, $\times 8$-OK and Elev-OK (level 43, index 0.59). The similarity tree for the Greek population (Annex 3) is quite similar but, for the same variables the two blocks are distinct: on the one hand, the type of tasks SP and on the other hand the others. SP is the only type of tasks which is worked, by Greek population, in $R_a$ registers and the other types of tasks are worked with the help of a conversion to base ten.

5.2 Not worked type of tasks

On the other hand, successes are weak for the type of tasks Elev because this type of tasks is very little worked. Greek students present more difficulties with this type of tasks (Gr: 20% NR and 22% OK ; Fr: 15% NR and 50% OK). This is without doubt a teaching effect. If, in the two institutions, several type of tasks have been worked in $R_a$ with the elaboration of treatment techniques, it is worth noticing that in Greece this type of tasks have been worked also by a treatment in base ten after a conversion.

This is useless for Elev which is a conversion $R_{ten} R_{eleven}$.

This table gives precisions for the use of base ten in the two teaching institutions.

<table>
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<th>$\times 8$</th>
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<th>EOb</th>
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<tr>
<td>Greece</td>
<td>« ten »</td>
<td>77%</td>
<td>78%</td>
</tr>
<tr>
<td></td>
<td>« NR »</td>
<td>16%</td>
<td>15%</td>
</tr>
<tr>
<td>France</td>
<td>« ten »</td>
<td>42%</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>« NR »</td>
<td>17%</td>
<td>15%</td>
</tr>
</tbody>
</table>

Resort to base ten is quasi-systematic for Greek students since the sum of percentages « ten » and « NR » is close to 100%. We find traces of these different roles of « ten » variables in the implicative graphs for the two populations (Annex 4, figures 13a & 13b). Whereas the « ten » variables are on top of the graph, next to « poly », « A » and « 1_10 » variables for the French population, they were found at the bottom of the graph for the Greek population.

For Greek students, it is the success of a type of tasks – for EO and $\times 8$ – which implies a base ten treatment whereas for French students, it is the use of base ten as a base for treatments which implies the success of the types of tasks.
6. Discussion on the development of R-praxis

Duval states (1993, 1996) that, it is important to have several representation registers in order not to mix up a mathematic object with one of its representations. This has been put in evidence by the type of tasks Elev when the number ten is very often mixed up with its code 10 in base ten.

For some students, we feel a tension between the technique in base ten and the coding of a number. Some students stay to the stage of astonishment but others write clearly that a number is odd in base seven and even in base ten (see figure 8). This problem comes from an elaborate R-praxis on a naturalized register, $R_{ten}$.

**Figure 8:** F122 from the Main test, the parity of a number depends on the coding base

Duval (1993, 1996) equally argues that it is important to be able to choose one register rather than another to carry out the treatments. It seems that the possibility of this choice masks one difficulty. In fact, and Greek population testifies this, the possibility to make a treatment in base ten systematically does not permit developing the R-praxis into a base other than ten.

It appears from our study that the choice of the register for treatments strongly depends on the type of tasks. These students who made an $R_{ten}$ treatment after a conversion by $\tau_{pd,ten}$ (possibly followed by a reconversion, see figure 5a) are not blocked by $R_{a}$, but they either lack treatment in this register, or they dare not to have an “adventure” into this register by lack of insurance. In both cases, there is no R-praxis made for the types of tasks SP and EO and it is necessary to build techniques. These are obtained by a conversion into $R_{ten}$ where one has techniques.

It seems that it is important to work on bases other than ten and also it is necessary to develop the associated R-praxis, without which the new registers risk to leave incomplete the cognitive role attributed to them by Duval. For doing this, and as we have explained in section 5.3, the technological role played by $R_{poly}$ register in the construction of numerical R-praxis seems particularly important.

7. Conclusion

Our first objective was to find the cognitive distinction between treatment and conversion made by Duval for the registers of numeration in base other than ten. In fact, during our first experimentation (Nikolantonakis & Vivier 2009) we could not put this fact in evidence. The Statistical Implicative Analysis (SIA) has been an indispensable tool by showing similarities between different variables of treatment on the one hand and of

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3 Duval (2006: 127) discusses this point saying that “it is the choice of treatment that makes the choice of register relevant”. We claim that this choice depends on types of tasks and R-praxis.
conversion on the other. SIA has permitted us to validate our hypothesis by making it precise since it is the distinction between base ten and the other bases of numeration that we put in evidence.

More precisely, we point out two cognitive difficulties particularly important for coding numbers into the position numeration:

- the introduction of representation registers \( R_a \) with \( a \neq 10 \);
- the introduction of representation registers \( R_a \) with \( a > 10 \).

These cognitive difficulties relative to \( R_a \) registers, are obviously connected and seemed, after our study, stable from one institution to the other and from one teaching to the other.

Nevertheless, the interpretation needs to go much further because registers and types of tasks influence jointly the mathematical activity. We can see that particularly in the second difficult item, EOb, which introduce one register \( R_a \) \( a < 10 \), but no internal technique to \( R_a \) is available. In that sense we have proposed one hierarchy to R-praxis in section 5.2.

It has to be noticed that if success percentages give some indications on this hierarchy, it is the implicative trees which were crucial for the elaboration of this R-praxis Classification. This point seems particularly rich from a methodological point of view. The theoretical framework of registers and praxis permits studying some key indicators which, by using SIA, show some relations and in return, justify the validity and the interest of our didactic analysis framework. The a priori analysis has shown the importance of polynomial writing register which constitutes a very efficient register for treating the types of tasks for which we have not disposed of an institutional technique. Unfortunately, not many students use the technological potential of this register. It is used in majority as a transitory register during a stage of conversion between one base \( a \) and the base ten. We have nevertheless validated our second hypothesis formulated from our previous analyses on the technological potential of polynomial writing, notably with similarities concerning two different types of tasks. The SIA permits us to go much further by showing, by the quasi-implications, that this \( R_{poly} \) register is important for the success of the types of tasks.

**References**


**Annexes**

**Annex 1: Tests’ items and results of preliminary study**

**SP:** find the Successors and Predecessors of the following numbers.

<table>
<thead>
<tr>
<th>SPa</th>
<th>SPb</th>
<th>SPc</th>
<th>SPd</th>
<th>SPe</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR OK</td>
<td>NR OK</td>
<td>NR OK</td>
<td>NR OK</td>
<td>NR OK</td>
</tr>
<tr>
<td>FR (26)</td>
<td>2 (8%)</td>
<td>15 (58%)</td>
<td>2 (8%)</td>
<td>13 (50%)</td>
</tr>
<tr>
<td></td>
<td>3 (12%)</td>
<td>15 (58%)</td>
<td>3 (12%)</td>
<td>16 (62%)</td>
</tr>
<tr>
<td>GR (100)</td>
<td>16 %</td>
<td>73 %</td>
<td>16 %</td>
<td>47 %</td>
</tr>
<tr>
<td></td>
<td>17 %</td>
<td>61 %</td>
<td>16 %</td>
<td>45 %</td>
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<tr>
<td></td>
<td>17 %</td>
<td>64 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**EO:** among the following four numbers, find even and odd numbers.

<table>
<thead>
<tr>
<th>EOa</th>
<th>EOB</th>
<th>EOc</th>
<th>EOD</th>
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</thead>
<tbody>
<tr>
<td>NR OK</td>
<td>NR OK</td>
<td>NR OK</td>
<td>NR OK</td>
</tr>
<tr>
<td>FR (26)</td>
<td>3 (12%)</td>
<td>17 (65%)</td>
<td>4 (15%)</td>
</tr>
<tr>
<td></td>
<td>17 (65%)</td>
<td>3 (12%)</td>
<td>18 (69%)</td>
</tr>
<tr>
<td>GR (100)</td>
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<td>64%</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>66%</td>
<td>15%</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15%</td>
<td>72%</td>
</tr>
</tbody>
</table>

**Annex 2: Tests’ items and results of main study**

A) Without justifying your response:

1) Find the successors and predecessors of
   a. \((66)_7\)    b. \((10100)_2\)  c. \((504302155)_6\)
2) Write numbers from twenty to thirty on base eleven
3) Arrange in growth order the following numbers
   \((303)_4\); \((203)_4\); \((1003)_4\); \((33)_4\)

B) By justifying in brief your response

1) Write the result of the multiplication of the number \((3405)_8\) by eight
2) Among the following numbers, find even and odd numbers
   a. \((65474)_8\)    b. \((623004261)_7\)

Coding items in main test

A1 – Successors and Predecessors: type of tasks SP

NR : Non Response
OK : correct solution
res IB: result on Initial Base
Ten: conversion on base ten for a treatment on base ten
erConv: at least one error into a conversion
erTreat: at least one error into a treatment

A2 – Coding on base eleven: type of tasks « Elev »
NR, OK.
Conf: confusion number/code of a number
« A »: use of a letter (or another sign).
« 1_10 »: use of the code 10 for number ten (except the answer (110)_{eleven}).

A3 – Classification: type of tasks « Class »
NR, OK, Ten.

B1 – Multiplication by eight: type of tasks « ×8 »
NR, OK, res IB, Ten, errConv, errTreat.
Poly: polynomial writing without calculation and treatment on this writing
« 27240 »: use of multiplication technique of the base ten on base eight

B2 – Even/Odd: type of tasks « EO »
NR, OK, Ten, errConv, errTreat, Poly.
Tech ten→IB: illicit use of the technique in ten base in initial base

<table>
<thead>
<tr>
<th>Class</th>
<th>SPa</th>
<th>SPd</th>
<th>SPc</th>
<th>SPb</th>
<th>EOa</th>
<th>×8</th>
<th>Elev</th>
<th>EOb</th>
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<tbody>
<tr>
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<td>7%</td>
<td>10%</td>
<td>10%</td>
<td>15%</td>
<td>16%</td>
<td>17%</td>
<td>19%</td>
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<tr>
<td>OK</td>
<td>82%</td>
<td>75%</td>
<td>85%</td>
<td>76%</td>
<td>67%</td>
<td>66%</td>
<td>61%</td>
<td>39%</td>
</tr>
</tbody>
</table>

Annex 3: Similarity trees of sub-populations

Figure 9: French
Figures 10: Greek

Annex 4: Extracts of implicative graphs
Figure 11, FR and GR: level 99: NR form the represented sub-graph whereas the sub-graph of OK makes other variables apparent.
Figure 12, FR and GR: level 100 in red, 99 in blue and 98 in green: the rest of the graph is that of the figure 9
Figures 13a, FR, and 13b, GR: levels 99 in red; 98 in blue

Brief Biographies

Laurent Vivier
Laurent Vivier has had a PhD in Mathematics in 1998. He worked as associate professor for 11 years in Institutes for Teachers’ Training. He is, since 2011, an associate professor in Mathematics Education at the University of Paris Diderot, France, and member of the Laboratoire de Didactique André Revuz (LDAR).

Konstantinos Nikolantonakis
Konstantinos Nikolantonakis has had a PhD in Epistemology and History of Mathematics in 1998. He is working actually as Assistant Professor on Mathematics Education in the Department of Primary Education at the University of Western Macedonia.